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PETER O. MINOTT
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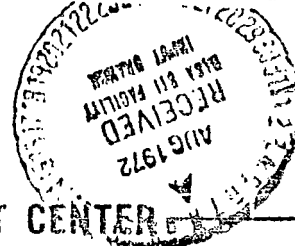MICHAEL W. FITZMAURICE

MARCH 1972

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

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Greenbelt, Maryland

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## ACKNOWLEDGEMENTS

iii

# TABLE OF CONTENTS

TABLE OF CONTENTS (Cont)

SECTION 7

CONCLUSIONS

APPENDIX A

UPPER A.R DATA

APPENDIX B

EVALUATION OF LOG-AMPLITUDE
CORRELATION COEFFICIENT

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*Data processed only for the argon wavelength.

# 1. INTRODUCTION

The intensive study of electromagnetic wave propagation through turbulent mediums began in the early 1950's. The first work of significance was performed in Russia by Kolmogorov[1], Chernov[2], and later Tatarski[3]. Although a great deal of work in this area was performed in the United States, intensive study did not begin until the early 1960's. One of the major driving forces which spurred interest in wave propagation through a turbulent medium was the advent of the laser, with its promise of data transmission bandwidths many orders of magnitude higher than possible with the highest frequency microwaves.

Although initially there was some interest in high-data-rate transmission between points on the earth's surface, it soon became apparent that the effects of weather and the line-of-sight nature of laser transmission would preclude high-data-rate laser communications except over short distances and under favorable weather conditions. The situation, however, for ground-to-space and space-to-ground communications is much more favorable, and therefore, work on this type of data link has been actively pursued by both NASA and the Air Force. Studies[4] have shown that by the use of redundant earth-based transmitter/receiver sites, link outages can be reduced to a few tenths of a percent. Through the use of airborne transmitter/receiver systems, even these minor outages can be eliminated.

From the beginning of the study of atmospheric turbulence, the sophistication of the theoretician has far outpaced the experimentalist. There are good and sufficient reasons for this state of affairs. The mechanism for the generation of turbulence is beyond the control of the experimenter and, therefore, it is impossible to perform controlled experiments in which the state of turbulence can be fixed while measurements are performed. The problem is further exacerbated by the fact that the experimenter rarely has sufficient instrumentation to even specify the state of the atmosphere during his measurements.

Some of the earliest experiments in optical propagation were made by astronomers, who were concerned with the variations in seeing and scintillation because these effects masked their ability to study the characteristics of extraterrestrial sources. Although considerable time and effort was expended in their studies, most of their work was performed during a period when no adequate theory was available to explain the mechanism of the observed effects, and therefore much of the work is of a subjective nature

with inadequate supportive data to interpret the results. Notable exceptions to this state of affairs are the work of Mikesell and Hoag[5], Protheroe[6], Young[7], Ramsey[8], Coulman[9], and later, Bufton[10]. Except for the studies made using astronomical sources, nearly all of the experimental attempts to verify theory have been made over horizontal propagation ranges. If a review of the proportion of work performed on horizontal ranges compared to vertical propagation paths were made, it would reveal that while thousands of experiments have been performed over horizontal ranges, almost nothing has been done to measure the effects of atmospheric turbulence on a laser beam propagating over a vertical path.

When work on the study of atmospheric turbulence was started at GSFC, by the authors, it became apparent after a short amount of study that the theoretical predictions of turbulence effects were being adequately pursued by other agencies and organizations and that the area which was most in need of investigation was the experimental verification of existing theory. Because all of the theoretical predictions of the optical effects are based upon the state of atmospheric turbulence, a development program was started to build a sensory device capable of directly measuring the state of turbulence. The outcome of this effort was the thermosonde, a lightweight electronic device capable of being borne aloft by a small weather balloon to measure the state of turbulence at altitudes to 100,000 feet, well above the last significant turbulence. In 1968, interest in laser propagation was at its peak, and many groups were considering methods of measuring turbulence in the upper atmosphere. Because of their ability to reach high altitudes and because they generate no atmospheric turbulence, the authors were at this time considering measurements made from high-altitude research balloons. Because of our interest in balloons as a vehicle for turbulence measurements, the Office of Advanced Research and Technology forwarded to us a proposal by the Sylvania Electronic Systems, Western Division to perform a series of balloon-borne laser propagation experiments. The outcome of this proposal was the experiment described in this report.

The experiment, which was titled "Balloon Atmospheric Propagation Experiment" (BAPE), was performed at White Sands Missile Range in the fall of 1970, and is to this date the only controlled measurement of laser propagation over a vertical path. The experiment was repeated in 1971 at the same location and the two flight series have been named BAPE I and BAPE II to distinguish between them. This report will describe only the data obtained in BAPE I.

The experiment plan was to measure the optical effects of turbulence over a vertical path while at the same time measuring, with the thermosonde, the distribution of turbulence along the path. In this manner, the optical effects predicted by theory could be compared to the measured results in an effort to verify the validity of theory. The experiment has not uncovered any phenomena in contradiction to existing theory, but has confirmed many of the existing predictions. We feel that although some new and unexpected phenomenon might perhaps be more dramatic, our confirmation of existing predictions is actually more constructive.

To this date there have been only three successful attempts to measure the characteristics of laser propagation between ground and space. All of these have been performed by the authors, starting with the GEOS II Laser Detector Experiment and followed by the BAPE I and BAPE II experiments. The results of these experiments confirm the predictions of existing theory within the limits of the experiment. Because of the importance of this field of investigation to the development of the laser as a communications tool and because of the demonstrated success of balloon-borne instrumentation as a means of obtaining atmospheric propagation data, future experiments of this type are being planned.

The balloon-borne experiment described in the subsequent sections of this report constitutes the first definitive set of laser data on the effects of atmospheric turbulence over a vertical or near-vertical path. As such, the experiment provides the basis for meaningful predictions of laser system performance and data which establishes the utility and limitations of various propagation theories. Therefore, this was an important experiment with far-reaching consequences.

Basically the objective of the experiment was to propagate two laser beams, one argon ($5145A^{0}$) and one $CO_2$ ($10.6\mu$), over a vertical path from the ground to receivers located above the atmosphere, and to measure the scintillation of the upgoing beams. Simultaneous measurements of the state of turbulence along the path allow the scintillation statistics to be compared on the basis of existing theory to the experiment results. This type of experiment has four important results:

1. The measurements of scintillation are directly applicable to estimation of scintillation of earth-to-space laser links.

2. The measurements of the state of turbulence provide new data on the state of atmospheric turbulence, no direct experimental data on turbulence above 3.51 m (10,000 ft) was available prior to the experiment.

3. By comparing the actual scintillation statistics with those based upon theory plus the turbulence profile, we can check the validity of existing propagation theories.

4. Knowing that these theories are correct (assuming that the theories check out), we can make far ranging predictions on the performance of laser communications systems with a high degree of confidence.

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## 2. SUMMARY

This section provides a brief synopsis of the results from the Balloon Atmospheric Propogation Experiment.

The vertical profile of turbulence in the first 6 km of the atmosphere was found to be approximately

$$C_N^2 = 4 \times 10^{-14} \, h^{-1/3} \, e^{-h/1000} \quad m^{-2/3}$$

The log-amplitude variance at $45^o$ from zenith had an average value of 0.198 at 0.5145 $\mu m$ and 0.00510 at 10.6 $\mu m$. Measured wavelength dependence of scintillation was found to be within 8% of that predicted by the 7/6 power law. The sample log-amplitude variance was itself a random variable with a log-normal distribution and a standard deviation of $\ln 2$. This indicates that the strength of turbulence varies with time in the same manner.

Log-amplitude covariance was measured and the correlation lengths were found to be, as predicted by theory, proportional to $\lambda^{1/2}$; the size of the correlation lengths was also found to be as predicted by theory. In order to test this last relationship, a method of approximating the atmosphere by a multilayer system of turbulence was developed and used to predict the correlation lengths. Both correlation length and the shape of the correlation coefficient predicted by this procedure show close correspondence to measured values.

Using the above method, aperture averaging for large apertures is computed and shown to be in close agreement with other methods of obtaining aperture averaging factors.

Autocorrelation of irradiance and power spectral density were measured and compared to predicted results. A close agreement between theoretical and measured values indicates that the temporal behavior of the scintillation can be obtained from a knowledge of the height of the turbulence layers and their wind speeds.

The results of the experiment represent a fairly complete investigation of the scintillation effects of atmospheric turbulence over a vertical path, although much remains to be measured. We have found no parameter which does not have the magnitude and behavior predicted by theory. Results of the experiment tend to confirm the validity of present predictions of the effects of atmospheric turbulence.

# 3. DESCRIPTION OF EQUIPMENT

## 3.1 BAPE PAYLOAD

The following sections describe the design and construction of the BAPE payload which was the flight vehicle used to carry the optical detectors of the experiment above the atmosphere. The payload was fabricated by Sylvania Electronic Systems Western Division under contract NAS 5-11679. The material from the following sections is in part abstracted from the BAPE Phase I report of August 14, 1969, with changes and additions made where required. The principal authors of this report were Mr. Arthur Kraemer, Dr. Paul J. Titterton, Mr. Tom Mulcahy, and Mr. Scott Overstreet. Other contributors include Mr. Clyde Brown, Mr. Connell Ward, Mr. Mickael Halus, and Dr. Douglas Woodman. Professor J. Richard Kerr of the Oregon Graduate Center acted as a program consultant.

The purpose of the material in the following sections is to familiarize the reader with the construction and operation of the payload so that he may better understand how the data was obtained and its significance.

### 3.1.1 GENERAL DESCRIPTION OF PAYLOAD

The BAPE payload shown in Figures 3-1 and 3-2 consisted of an optical receiver system in an inverted azimuth-elevation gimbal system. The elevation axis (see Figure 3-3) contains four optical receivers, a star tracker, servo electronics, and two cube corner retroreflectors. It is pivoted on bearings supported by the azimuth gimbal and driven by a dc torquer through elevation angles from -26 to -90 degrees. The azimuth gimbal supports the elevation axis and connects it to the azimuth axis. The large box-shaped section of the azimuth gimbal contains batteries, telemetry encoders, and other electronics for the operation of the servo system. These electronics are connected by a cable wrap to the elevation axis. Above the azimuth gimbal is the azimuth bearing and slip ring assembly which provides continuous rotation in the azimuth direction and electrical connection to the gondola. A dc torquer, tachometer, clutch, and azimuth encoder are attached to the top of the azimuth bearing and slip ring assembly to complete the azimuth drive system. The gondola (see paragraph 3.1.5) provides a framework for supporting the components attached below the azimuth axis and for supporting the battery boxes. It consists of three arms connected at their inner ends to the elevation axis and at their outer ends to the battery boxes. The battery boxes contain the

SUPPORT CABLES

AZIMUTH DRIVE SERVO

AZ
AXIS

SLIP RINGS AND
AZIMUTH BEARINGS

BATTERY BOX

AZIMUTH GIMBAL

EL
AXIS

CRUSHABLE
CRASH PAD

MOVABLE MIRROR

STAR TRACKER

FIXED DETECTOR

$CO_2$ CUBE CORNER

ARGON CUBE CORNER
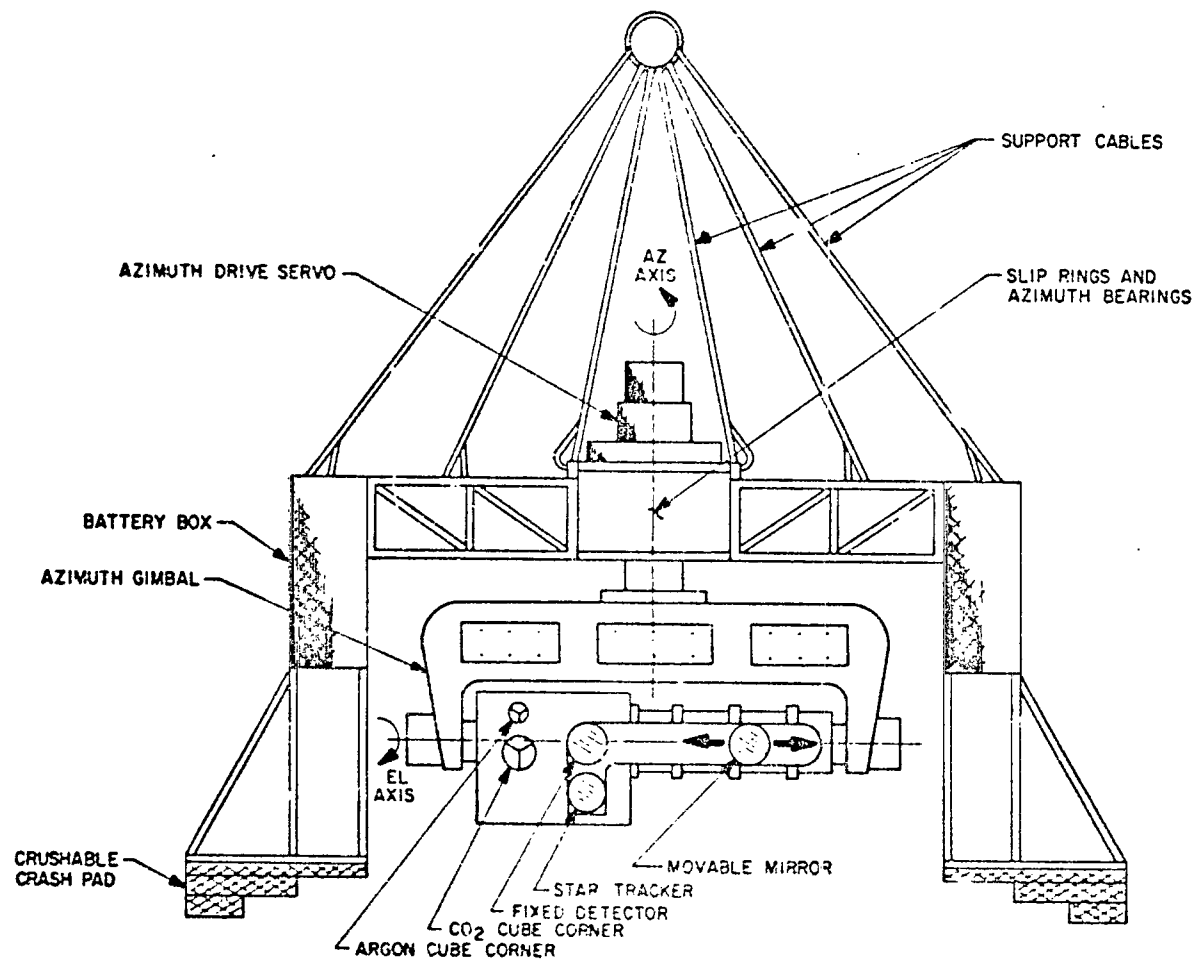
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Figure 3-1. BAPE Payload
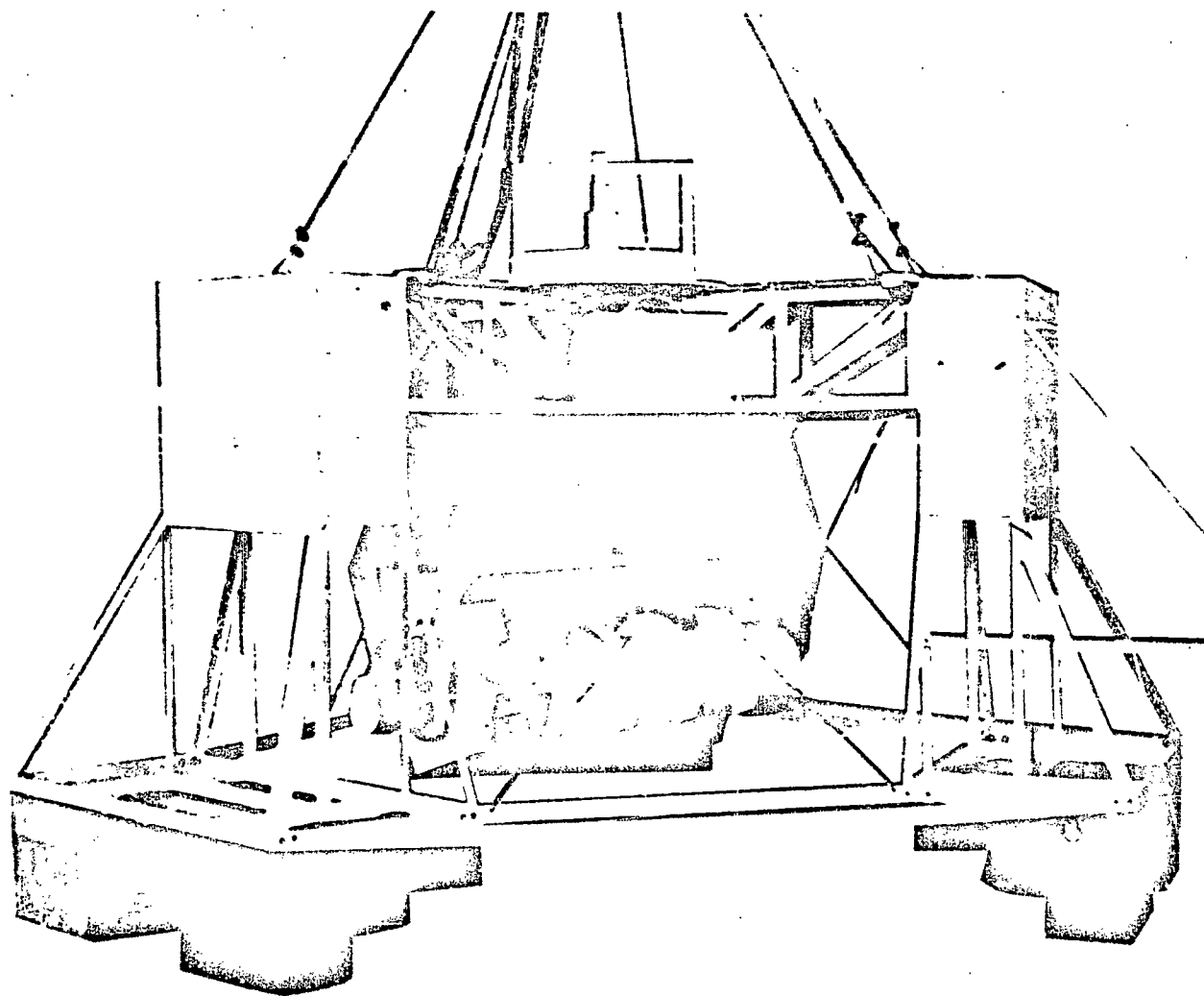
Figure 3-2. BAPE Payload
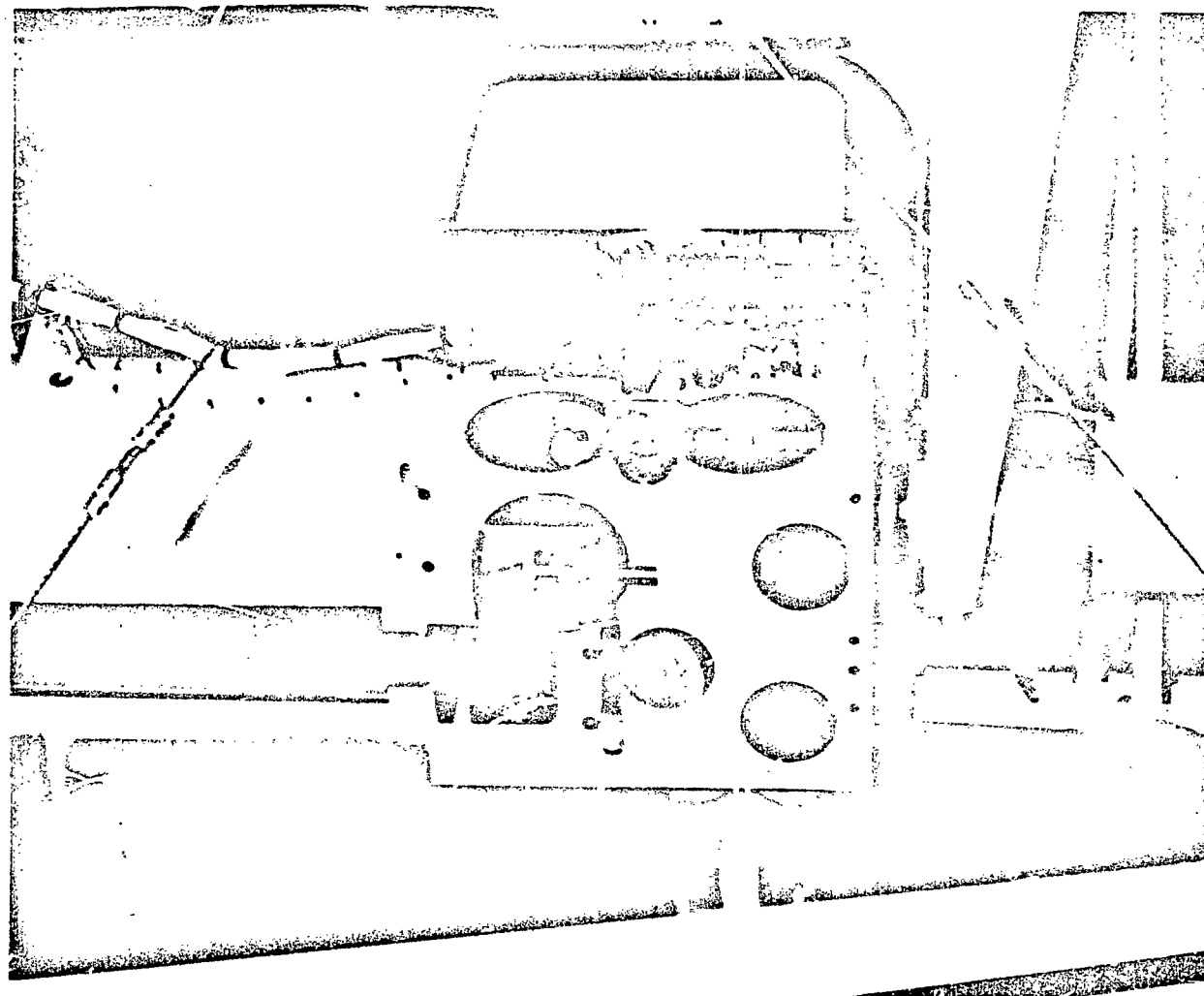
Figure 3-3. Elevation Axis

batteries necessary to power the payload as well as telemetry and electronics systems. The mass of the heavy batteries on the ends of the arms acts as an inertia flywheel against which the azimuth axis drives. Struts connected to the bottom of the battery boxes provide protection of the payload when it strikes the ground at the end of the parachute descent. Disposable, corregated cardboard crush pads on the bottom of these struts are used to absorb the energy of the impact.

## 3.1.2    ELEVATION AXIS AND OPTICAL DETECTOR SYSTEMS

An optical schematic of the elevation axis and optical detector systems is shown in Figure 3-4. The elevation axis contained two detectors at each of the wavelengths 0.5145 $\mu$m and 10.6 $\mu$m. One system operated fixed and the second system operated with a movable input mirror. In system I, the received beam was received by the dichroic beam splitter, which was used to separate the argon (0.5145 $\mu$m) and the $CO_2$ (10.6 $\mu$m) beams. The dichroic reflected the 10.6-$\mu$m beam through a refractive f/1 germaniur. lens onto the HgCdTe detector mounted in a liquid nitrogen dewar. The 0.5145-$\mu$m beam passed through the dichroic and was reflected through a lens arrangement onto the photomultiplier detector.

In system II, the received beam was reflected off the movable input mirror and a fixed mirror prior to reaching the dichroic beam splitter. The optical arrangement for system II was identical to system I after the dichroic beam splitter. Additionally, a quadrant star tracker with an f/3 lens was located in the elevation axis.

Figure 3-5 shows the exterior of the elevation axis.

The movable mirror varied its position from 4 cm to 100 cm with respect to the fixed system. The movable mirror traveled on precision stainless steel rods using Thomson ball bushings to reduce frictional loading, and was driven by a gear head servo motor. The location of the movable mirror was controlled by an electrically operated system with readouts at the ground station to indicate mirror positions.

The electronic system for the optical receivers was designed to extract the atmospheric modulation on the laser beam and convert it to an analog signal of 1-kHz bandwidth. Since it was desirable to observe low-level signals accurately, it was necessary to eliminate the effects of background. Therefore, the laser beam was chopped with a 50% on-off duty cycle at the transmitter at a 5-kHz rate and transmitted through the atmosphere to the receiver, thus acquiring the atmosphere modulation. The laser

ELEVATION AXIS SHAFT

ELEVATION AXIS FRAME

STAR TRKR

GERMANIUM LENS

NO1 CO2 DET

NO2 CO2 DET

GERMANIUM LENS

GERMANIUM LENS

LENS

NO1 ARGON DET

NO2 ARGON DET

STAR TRACKER OBJECTIVE

LENS

FIXED DICHROIC BEAM SPLITTERS

FIXED MIRROR

MOVABLE MIRROR
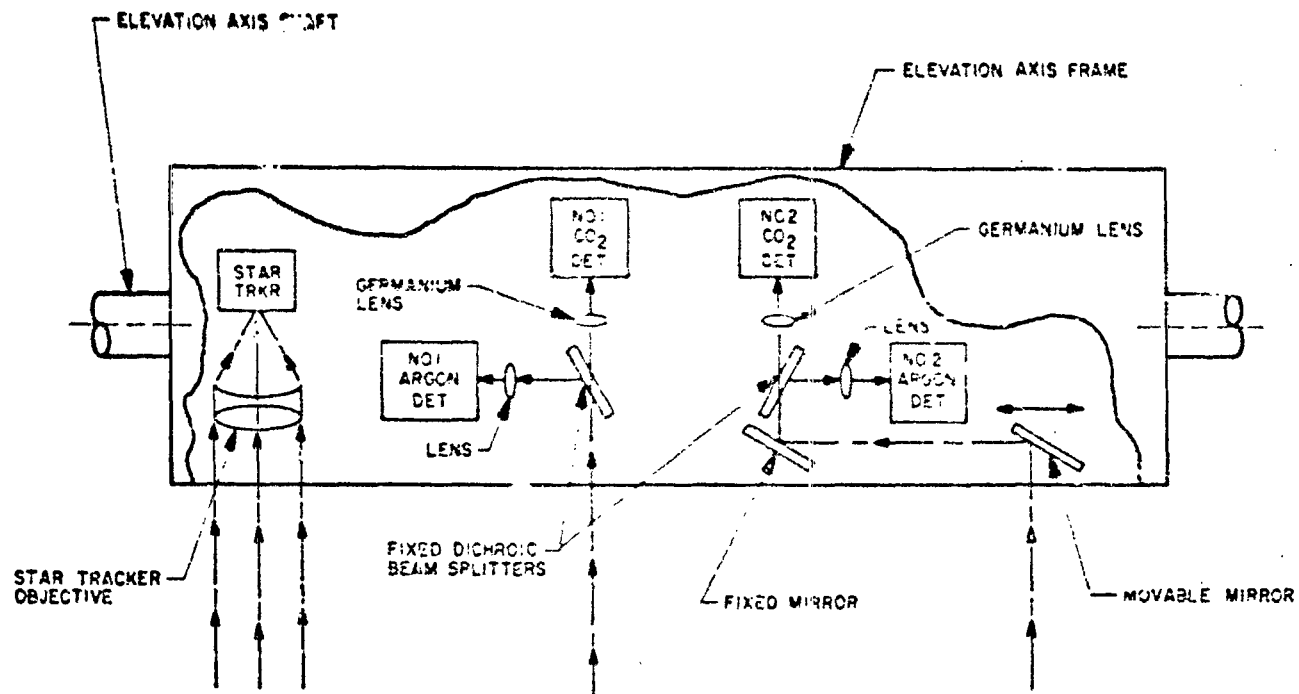
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Figure 3-4. Optical Schematic, Elevation Axis and Optical Detectors
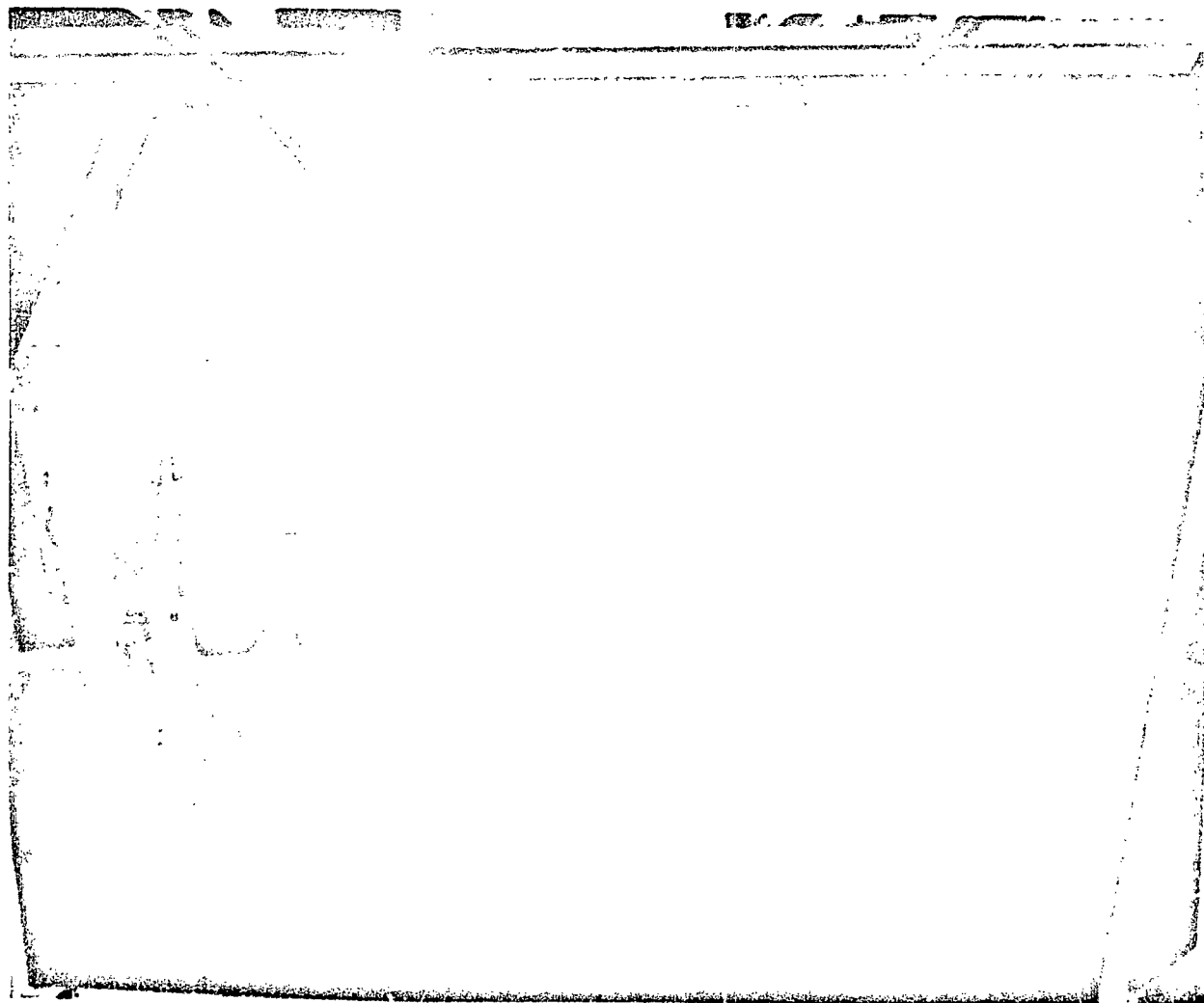
Figure 3-5. Elevation Axis, Exterior

receiver observed both the chopped laser beam (with atmospheric modulation) and a
slowly varying background and rejected the background with a high-pass filter. The
signal was returned to the ground (via the telemetry system). Figure 3-6 shows the
optical receiver electronic system.

### 3.1.3 QUADRANT STAR TRACKER

A quadrant star tracker mounted on the elevation axis was used to track the 0.5145-$\mu$m
laser beam from the ground transmitter to an accuracy of $\pm1$ milliradian. This infor-
mation was relayed to the attitude stabilization system to point the elevation axis with
its four optical receivers. Since the lens in front of the quadrant photodetector made
such a small image that the range of the star tracker was less than that required, it
was defocussed to provide a greater range of error signals.

The quadrant axes of the photodetector were positioned 45 degrees (see Figure 3-7)
from the elevation and azimuth axes so that the error signal for each axis generated by
differentially connecting the diagonal quadrants was

$$\Delta A = 2\sqrt{2}\, r\, \Delta x \qquad \text{for } \Delta x \ll r$$

for uniform energy density, where

$\Delta A \quad = \quad$ increase in spot area in quadrant 1 minus decrease in spot
area in quadrant 3

$\Delta x \quad = \quad$ misalignment along x axis

$r \quad = \quad$ spot radius

Positioning the instrument axes 45 degrees from the quadrant detector axes resulted
in no crosstalk between the error signals for small misalignments.

The 1-inch useful photocathode surface allowed acquisition of the ground transmitter
beam over a $\pm12.5$-degree field of view.
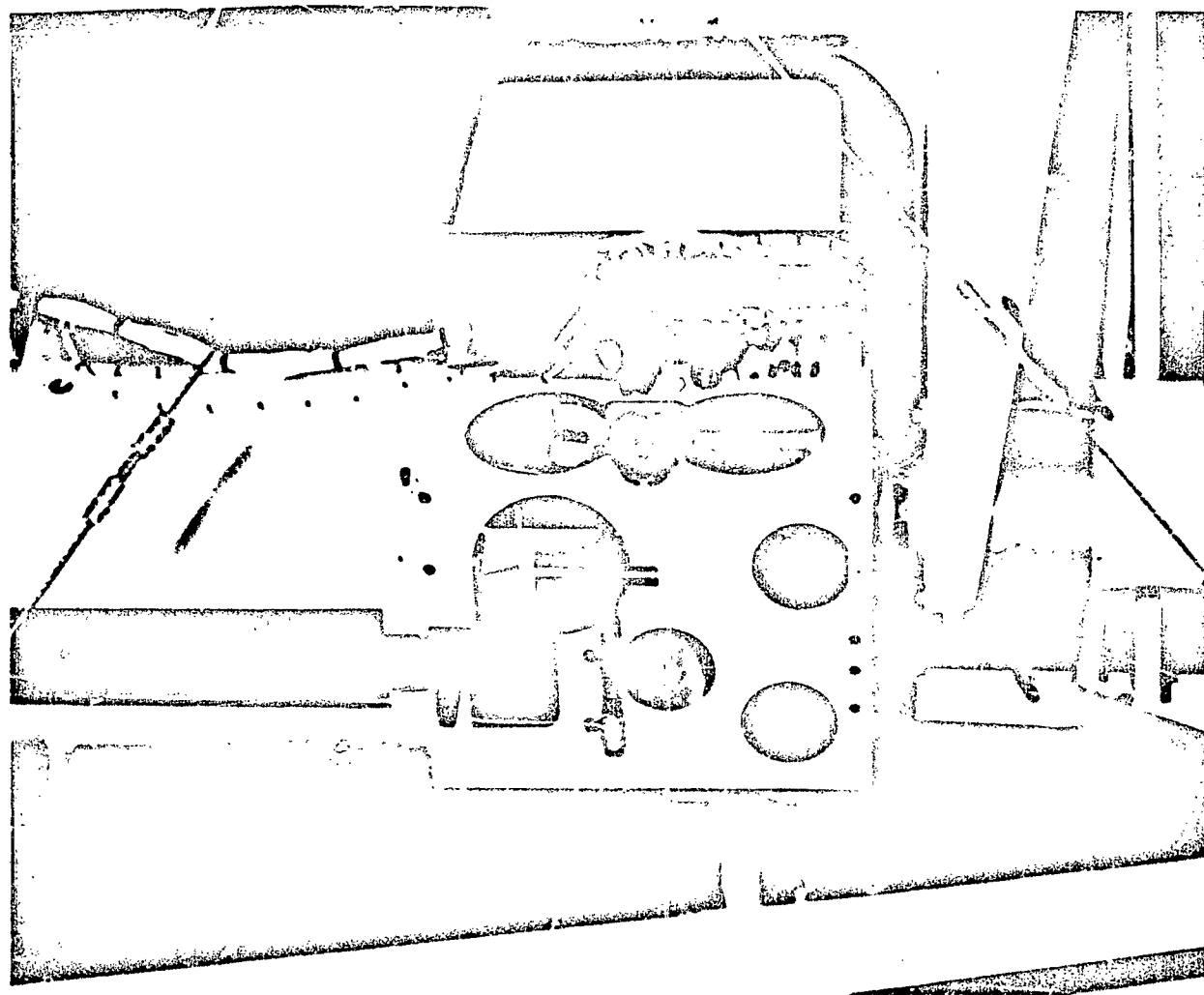
Figure 3-6. Optical Receiver Electronic System

SPOT CENTERED

$\Delta x$

Y'

X'

SPOT MISALIGNED
BY $\Delta x$

2

3

1

X

Y

4

$\dfrac{\Delta x}{\sqrt{2}}$

X, Y-INSTRUMENT AXES
X', Y'-QUADRANT DETECTOR AXES
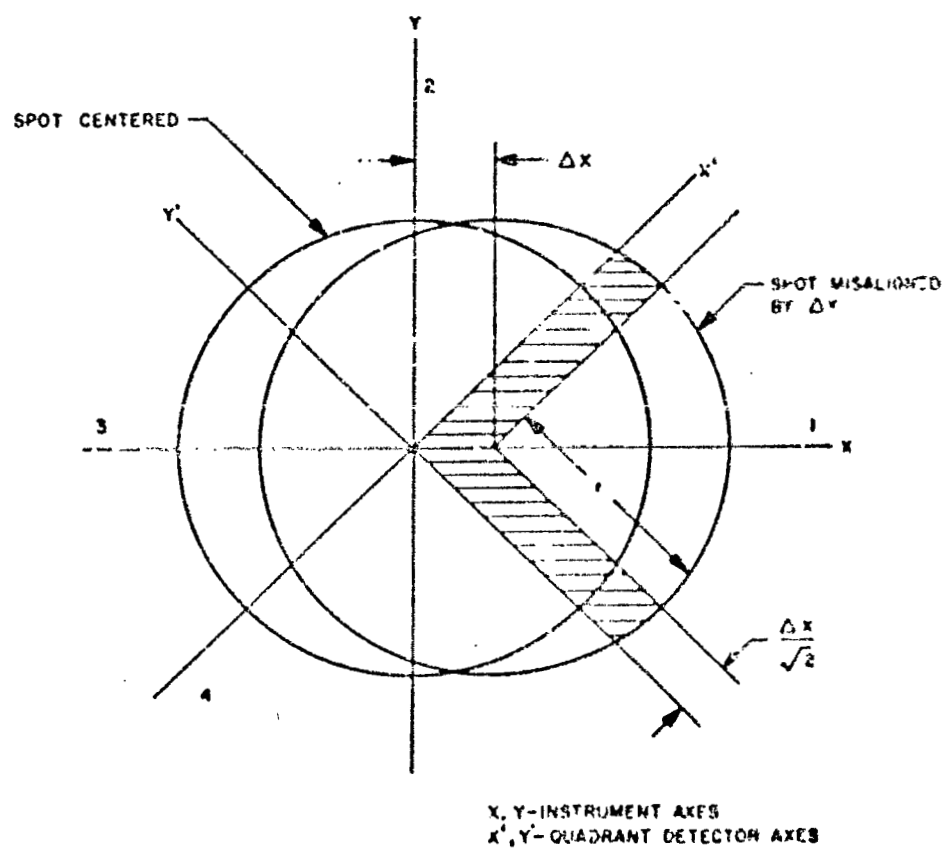
DIFFERENTIAL AREA $\approx 4 \bullet \dfrac{\Delta x}{\sqrt{2}}$, (QUADRANT I — QUADRANT 3)

Figure 3-7. Quadrant Detector Error Signal

### 3.1.4   ATTITUDE STABILIZATION SYSTEM

To keep the optical receivers continuously pointing at the upcoming beam, it was necessary to mount them within two controlled orthogonal axes of an elevation-under-azimuth configuration. These axes were controlled from the ground by way of the up telemetry link, and their positions were known at all times from data transmitted on the down telemetry link and displayed for the operator on the ground. Once the up-coming beam was acquired by the quadrant star tracker, it developed azimuth and elevation error signals which were amplified and applied to the axial drives in such a way as to maintain the instrument pointing at the beam. The positioning system is shown in Figure 3-8.

To acquire the beam initially, the operator utilized the elevation and true bearing data from the beam pointing equipment. He first commanded the elevation axis to depress by the same angle that the 0.5145-$\mu$m laser beam from the ground tracking system was elevated. Next, he commanded azimuth axis of the payload to look along a true bearing that was the reciprocal of the bearing from the ground tracking system to the balloon. When beam interception did not occur, the operator conducted a search through a small zone about this nominal position to effect intercept. An auto gate circuit automatically put the attitude stabilization system into the automatic tracking mode when the beam was detected. An override command was provided in order to break track on any un-desired energy sources.

The relationship of the gondola to magnetic north was measured with a flux gate trans-mitter of the type used in gating aircraft directional gyroscopes. For best accuracy and consistent operation, the device was located as far away from the rest of the equip-ment as possible. For this reason, it was placed at an outboard location on one of the gondola arms. Thus, the flux gate transmitter measured the relationship of the gondola with respect to the earth. The optical receiver system was positioned in azi-muth with respect to the gondola and the angle between the two was measured by a syn-chro differential transformer which was driven electrically from the flux gate trans-mitter. The output of the synchro differential was then the bearing of the optical re-ceiver system with respect to magnetic north. This output was converted to a dc analog voltage which fed a voltage-controlled oscillator (VCO) in the telemetry system. A servo repeater shown in Figure 3-8 accomplished this function.

Each axis was driven by a dc servo motor and amplifier combination. The amplifier inputs were either the amplified signals from the quadrant star tracker in the automatic tracking mode or a constant velocity (dc level) input that was applied when manual control was being exerted from the ground station. The ground controller could also override all other controls by actuating the standby mode. This mode was employed to break track from any undesired energy sources. At any time that control was not being exerted on the axes, they were held in position by electro-mechanical energize-to-release type brakes.

When the optical receiver system was positioned by either axis, the remainder of the equipment (including the balloon) reacted in the opposite direction. The amount of the reaction was proportional to the ratio of the moments of inertia of the optical receiver system and the remainder of the equipment. When the equipment reacted to an elevation change, the restoring force of gravity quickly restored the original orientation. The restoring forces in azimuth were less dependable and of lesser magnitude, however, consisting of wind effects and friction between the equipment and the surrounding air. To keep the apparatus relatively stable then, it was important to minimize the reaction to azimuth motion. This was accomplished by utilizing the gondola as a reaction wheel. As little equipment as possible and practical was contained within the positionable portion of the payload, and the remainder was packaged at the extremities of the gondola. In this manner it was possible to keep the ratio of the azimuthal moment of inertia of the parts below the azimuth axis lower than one-tenth that of the remainder of the equipment, a condition that assured relatively smooth operation. The arms of the gondola also served the purpose of protecting the centrally located optical receiver system (the most expensive part of the equipment) upon landing.

## 3.1.5 GONDOLA

The gondola (Figure 3-9) was the basic framework for supporting the remainder of the systems of the balloon payload. It consisted of an aluminum girder frame supporting the azimuth axis and slip ring at its center, and the three battery boxes at its extremities. In addition to its role as a framework and a container for the batteries, the gondola also acted as a reaction wheel for the servo system and as a protective cage for the more delicate systems of the payload. Because of its function as a protective cage, it normally sustained some damage in each flight and was therefore constructed as simply as possible to facilitate repair. Corregated cardboard crush pads below

EL POSITION DATA

TRUE BEARING
(MAGNETIC)

REFERENCE AC

REF TO THE
REACTION WHEEL

CT

S
AMPL

BATTERY

INVERTER
AND
FILTER

FLUX GATE
TRANSMITTER

CDX

DC POWER

MANUAL

POS

NEG

MANUAL DRIVE SIGNAL

LEFT COMMAND

AUTO

RIGHT COMMAND

A

AUTOTRACK OVERIDE

POS

NEG

MANUAL DRIVE SIGNAL

DOWN COMMAND

EL

UP COMMAND
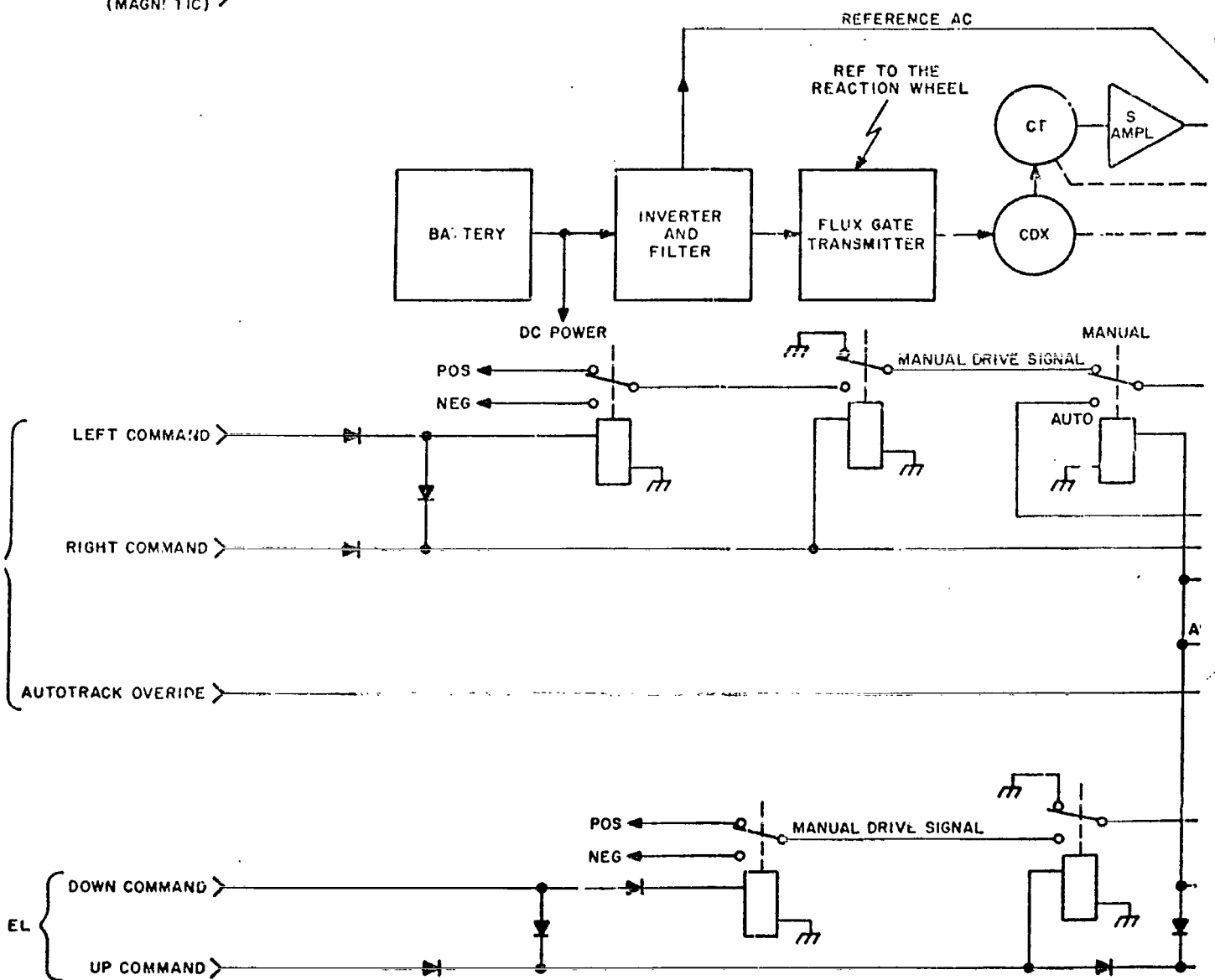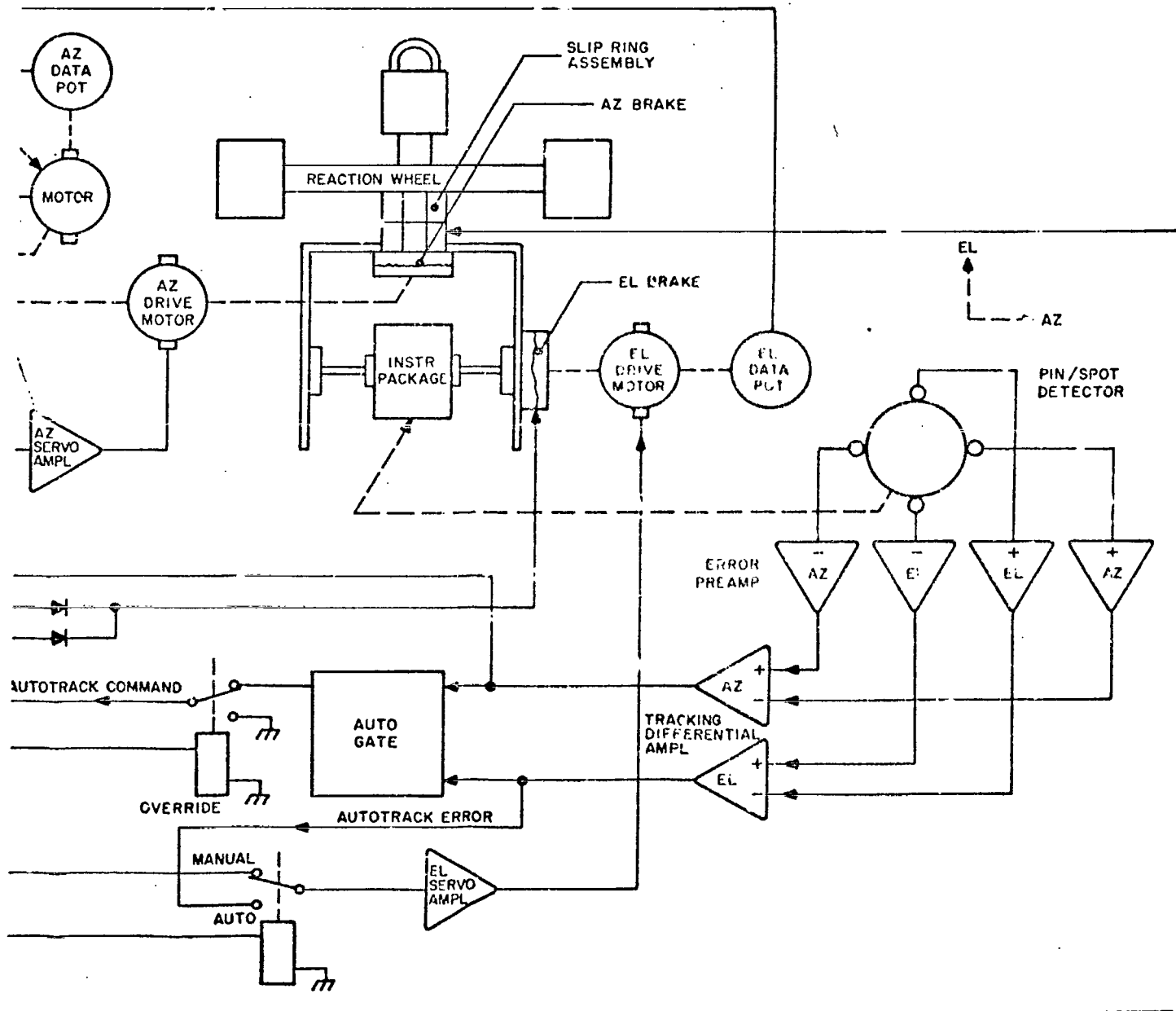
Figure 3-8. Positioning System

Figure 3-9. Gondola Framework

each battery box absorbed most of the landing impact and were replaced after each flight. The top of the gondola was equipped with a roll cage to protect the azimuth servo in the event of a rollover, and with sheet metal covers to act as sun shades as well as deflectors for ballast dropped from the balloon.

The gondola was basically a hexagonal parallelopiped with a height of approximately 5 feet and width across the flats of the hexagon of slightly less than 10 feet. It weighed (including the parts of the azimuth and elevation axes) approximately 750 lbs. Batteries were capable of operating the payload for 6 hours. The gondola was suspended in flight by nine steel cables (three from each arm) which were fastened to a steel O-ring approximately 15 feet above the azimuth axis (see Figure 3-1).

This method of support eliminates the bending loads which would be incurred by a two- or three-point support, and protects the payload from damage by the shock of the parachute opening.

## 3.2   THERMOSONDE

The optical measurements performed have value only for the ad hoc conditions present during the experiments unless the atmospheric conditions are known well enough to allow a full understanding of the turbulence effects and provide a real comparison with vertical propagation theory. This implies that not only the macro-meteorological variables such as wind speed and temperature as a function of altitude must be known, but that micro-meteorological variables such as the fluctuations in the temperature must also be known. The purpose of this section is to discuss the measurement basis, techniques, and apparatus for the fast temperature fluctuations.

For "clear" weather, the single parameter that describes the atmosphere's adverse effects on light propagation is the structure parameter of the index of refraction[1]. This structure parameter characterizes the index of refraction fluctuations and enters into the theoretical description of the atmosph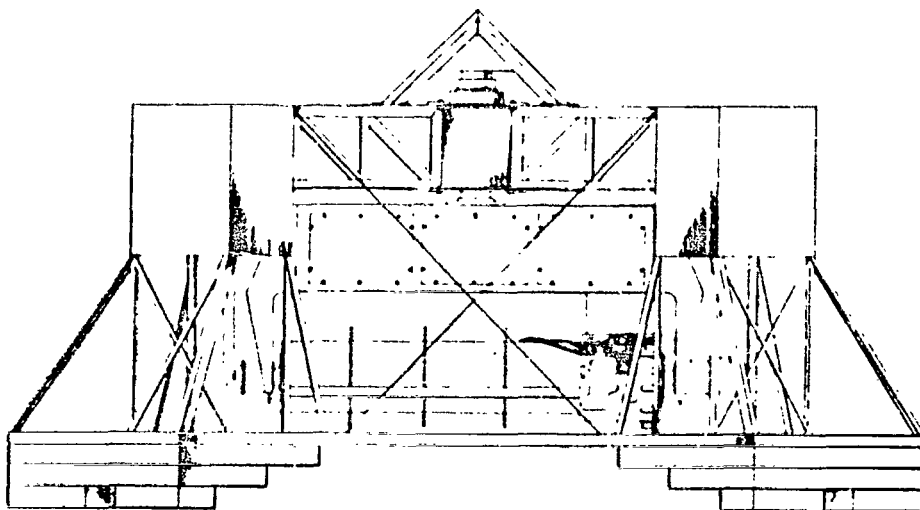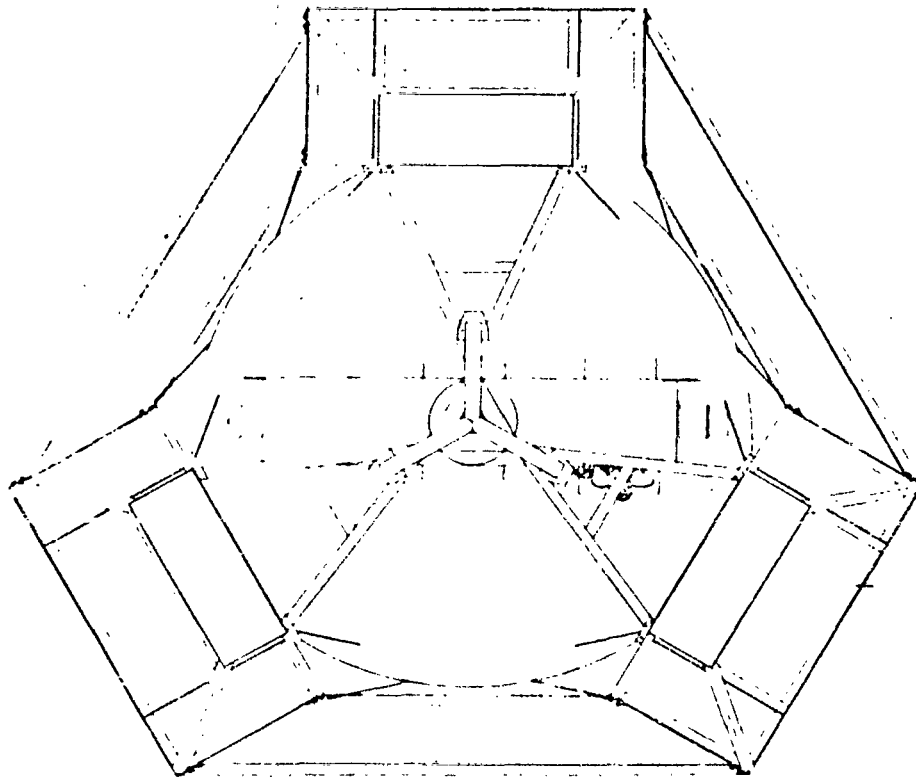ere's effects on light propagation[1-5]; that is, variance and covariance of scintillations, aperture averaging, transmitter aperture dependence, beam wander, beam spread, and the phase-front effects.

Being a property of a gas (air), the index of refraction fluctuations can depend on its constituents, its pressure, and its temperature. Experimentally, it has been shown that the only dependence on the constitutuents is via the density[6], which is well known. In addition, the pressure fluctuations are negligible[6] relative to those of the temperature. Therefore, the temperature fluctuations are directly related to those of the index of refraction.

3-16

Formally, the Obukhov-Kolmogorov theory[7] of turbulence states that the structure function of the index of refraction behaves as

$$D_N(\rho, h) \equiv \langle (N(x_1) - N(x_2))^2 \rangle = C_N^2(h) \, \rho^{2/3} \tag{1}$$

for $\rho \equiv x_1 - x_2$ and $C_N^2$ the index of refraction structure parameter, where h is the altitude, and is valid only with $l_o \ll \rho \ll L_o$, and the line joining the measurement points is perpendicular to the wind direction.

In like manner, the temperature structure function[7] is

$$D_T(\rho, h) = C_T^2(h) \, \rho^{2/3} \tag{2}$$

and[7]

$$C_N^2(h) = \left[ \frac{77.6P}{T^2} (1 + \frac{0.0075}{\lambda^2}) \right] C_T^2(h) \times 10^{-12} \tag{3}$$

for pressure (P) in millibars, temperature (T) in degrees K., and wavelength ($\lambda$) in $\mu$m. A measurement technique that suggests itself (and is widely used in ground systems[9-13]) is to mount two fast, sensitive sensors in a bridge configuration at a known distance apart. By suitably squaring and averaging the output, the temperature structure function is obtained. With $\rho$ known, equation (2) is used to obtain $C_T^2$ (h) and equation (3) is used to obtain $C_N^2$ (h), using density tables[8] and a simultaneous measurement of the temperature.

Sensitivity, speed, and uniformity of the two fast sensors was of prime importance. High signal levels were required to accurately measure $D_T^2$ and the sensors had to have matched temperature-resistance curves over the entire range of temperatures to be measured[14]. With thin-wire sensors, speed was no problem, and uniformity of length and cross-section was relatively easy to obtain. Sensitivity was the main questionable feature of thin-wire sensors.

The responsivity of a metallic wire sensor can be calculated from the applicable resistance-versus-temperature equation

$$R_T = R_0 (1 + \alpha T) \tag{4}$$

where $R_0 = \rho l / s$ ($\rho$ being the resistivity, $l$ the length, and s the cross-section) and $\alpha$ is the temperature-resistance coefficient.

The value $\rho$ for plat inum-rhodium[15] is of order $20 \times 10^{-16}$ ohms per cm, and $\alpha$ is of order[12] $1.3 \times 10^{-3}$ per $^{\circ}$C. Therefore, for a 1-mm long, 2-$\mu$ diameter platinum-rhodium wire, $R_o \approx 64\Omega$ and $\Delta R \approx 8.3 \times 10^{-2}$ ohms per $^{\circ}$C. Therefore, demanding a sensitivity of $5 \times 10^{-4}$ $^{\circ}$C means a resistance difference of 41.5 $\mu$ohms must be discernible.

Too much current cannot be fed through the wire because the self-heating will mask and distort the turbulence effects the wire is supposed to measure.
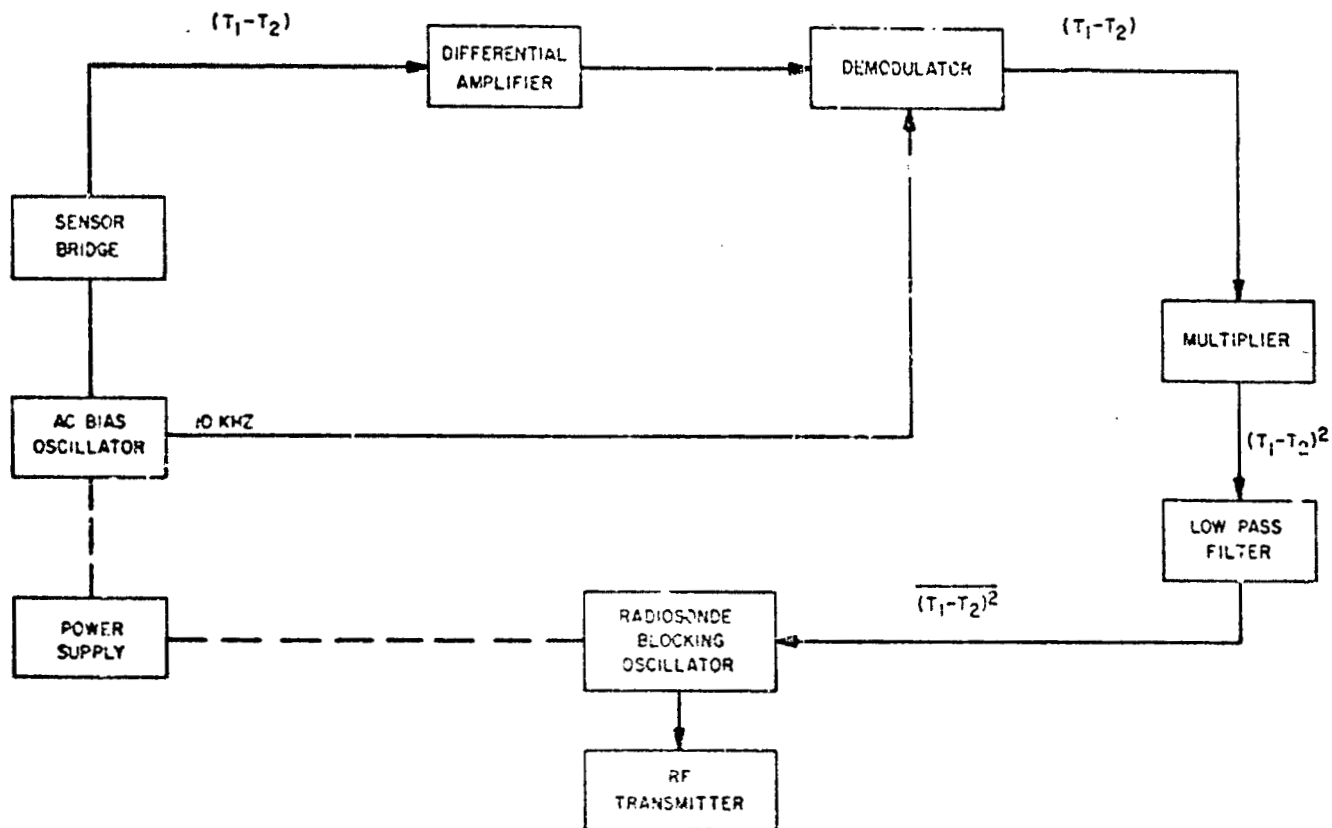
Therefore, with these design restrictions and the fact that a temperature difference was desired and amplifiers that do not extend down to de have much lower noise, the circuit used is as shown[16] in the block diagram, Figure 3-10.

The thermosonde is the box shown on the right hand side of Figure 3-13 suspended from the load bar. The rather large bulk is due to the large battery supply required to run the system for 8 hours and to keep it warm for that period. The actual circuitry and instrumentation weighed less than 20 lbs. In order to eliminate the effects of balloon wake in the $C_T^2$ measurement, the thermosonde was suspended from a reel hung from the load bar of the balloon. When the balloon reached an altitude of 330 m (1000 ft), the reel was released allowing the thermosonde to descend. A hydraulic drag mechanism in the reel limited the rate of descent to 17 ms$^{-1}$ (50 ft s$^{-1}$) giving a deployment time of approximately 20 sec. This deployment technique allowed error-free measurements to start at an altitude of about 60 m ($\approx$ 200 ft). The reel mechanism and the sonde can be seen in Figure 3-14.

## 3.3  FLIGHT VEHICLE

### 3.3.1  BALLOON CHARACTERISTICS

The balloon selected for the BAPE flights was a $2.0 \times 10^6$ ft$^3$ polyethelene balloon. This selection was based upon a requirement of an altitude of about 90,000 feet (27.4 km) and the gross weight of the BAPE payload ($\approx$ 750 lbs) plus the weight of Air Force balloon control and navigation equipment ($\approx$ 1000 lbs). The total weight of approximately 1750 lbs plus the altitude of 90,000 feet makes the $2 \times 10^6$ ft$^3$ balloon the proper choice for this type of flight (see Figure 3-11). Launch, navigation, recovery and all other aspects of the flight vehicle were handled by Detachment 1, Air Force Cambridge Research Laboratories — Balloon Research and Development Test Branch, Holloman Air Force Base, New Mexico, under the command of Lt Col Robert J. Reddy.

Figure 3-10. Preliminary Sensor Electronics Design, Block Diagram

1062 3.2-1

AFCRL POLYETHYLENE BALLOONS

NOTE
1 THIS CHART IS NOT ALL INCLUSIVE
2 USE FOR ESTIMATING ONLY
3 SOLID LINES ARE FLIGHT PROVEN
4 DASH LINES ARE UNDER DEVELOPMENT

GRAPH 1 — 1 JULY 70
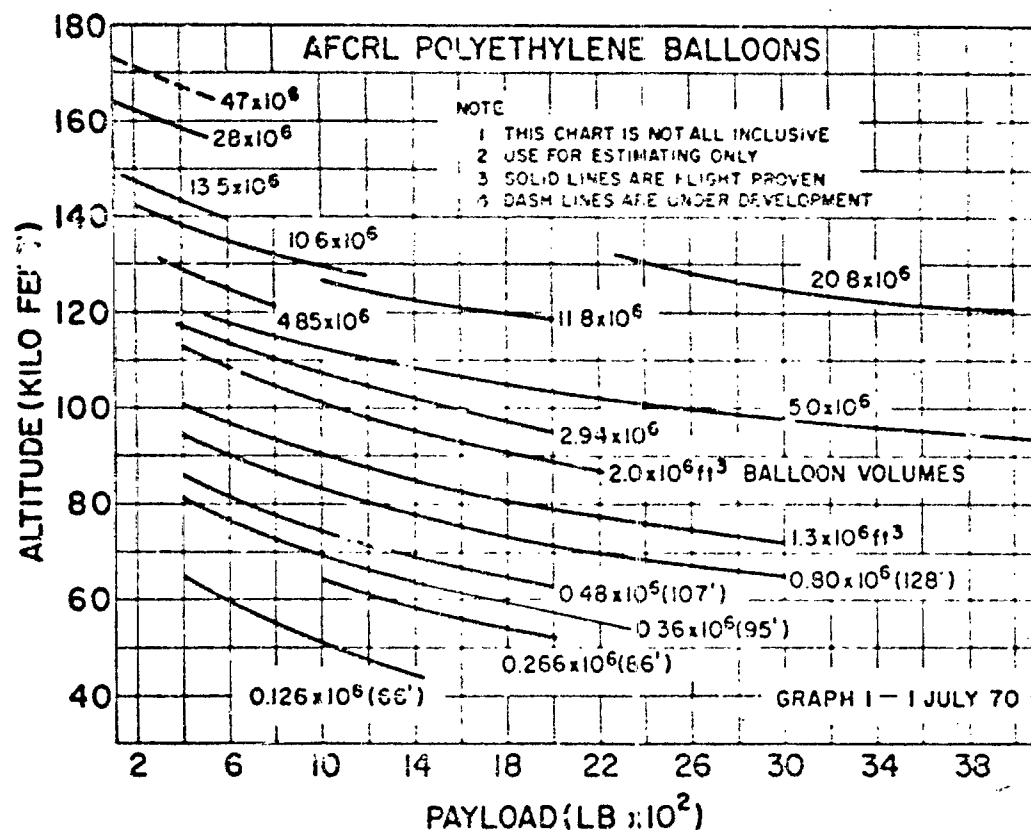
Figure 3-11.   Balloon Requirements

### 3.3.2 LOAD BAR AND RIGGING

The experiment flight configuration is shown in Figure 3-12. The BAPE payload is supported by nine steel cables which converge at a central point (Figure 3-13). This point is attached by an explosive shackle to the Air Force load bar which supports the Air Force navigation and flight control system as well as the thermosonde system and reel. In Figure 3-13 the components supported on the load bar are, from left to right: ballast hopper, command receiver and telemetry system, backup navigation transmitter, ballast hopper, and reel and thermosonde assembly. Flight configuration (reel not released) is shown in Figures 3-14 and 3-15*. The load bar, as shown in these figures, is supported by a second set of steel cables which converge on a ring at the bottom of the parachute shrouds. The load is supported on the shrouds of the open parachute and attached to the bottom of the balloon. The balloon is of the zero-pressure type, meaning that the bottom of the balloon is open and the helium is trapped only by its own buoyancy.

Vertical control of the balloon is controlled by the ballast hopper, shown in Figure 3-13, which drops iron filings to ascend, and a poppet valve about 15 inches in diameter (Figure 3-16) which releases helium to descend. Horizontal control of the balloon is obtained by ascending or descending into wind layers blowing in the desired direction.

Upon termination of the flight, the balloon is destroyed by an explosive device and the payload descends on the parachute (Figure 3-17). Upon impact the chute and load bar are disengaged from the payload by the explosive shackle so that ground winds do not drag the payload.

### 3.3.3 EXPERIMENT SITE AND FLIGHT PROFILE

The experiment was performed at the SCAT site on Holloman Air Force Base which is part of the White Sands Missile Range in New Mexico. The position of the site is shown in Figure 3-18. The site was on a level plain with semidesert vegetation approximately 7 miles northwest of Alamogordo, New Mexico. The plain is a valley floor with an altitude of 4000 ft (1.2 km) above sea level. The San Andres mountain range with peaks of about 8000 feet (2.1 km) lies approximately 30 miles to the west, and the Sacramento range with peaks to 11,000 feet (3.3 km) lies 10 miles to the east. During the early morning hours, cold air trapped in the valley during the night drains southward.

*The balloon in Figure 3-15 is a 60-ft-diameter balloon much smaller than the balloon used for the flights. It was used for tethered tests.

120' DIA

172'

242'

262'

RELEASE MECHANISM

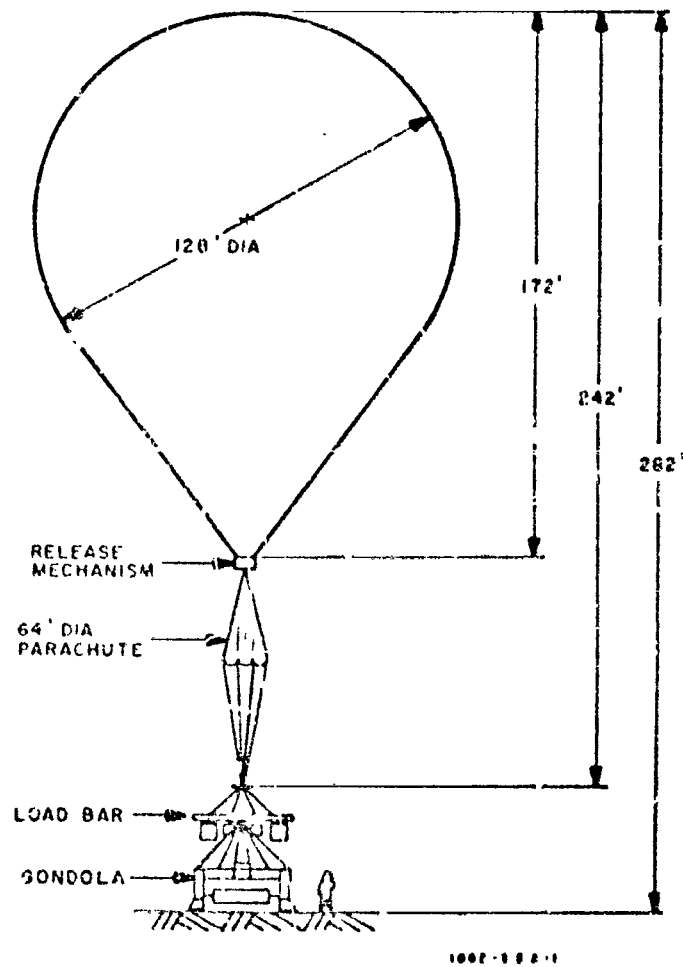64' DIA PARACHUTE

LOAD BAR

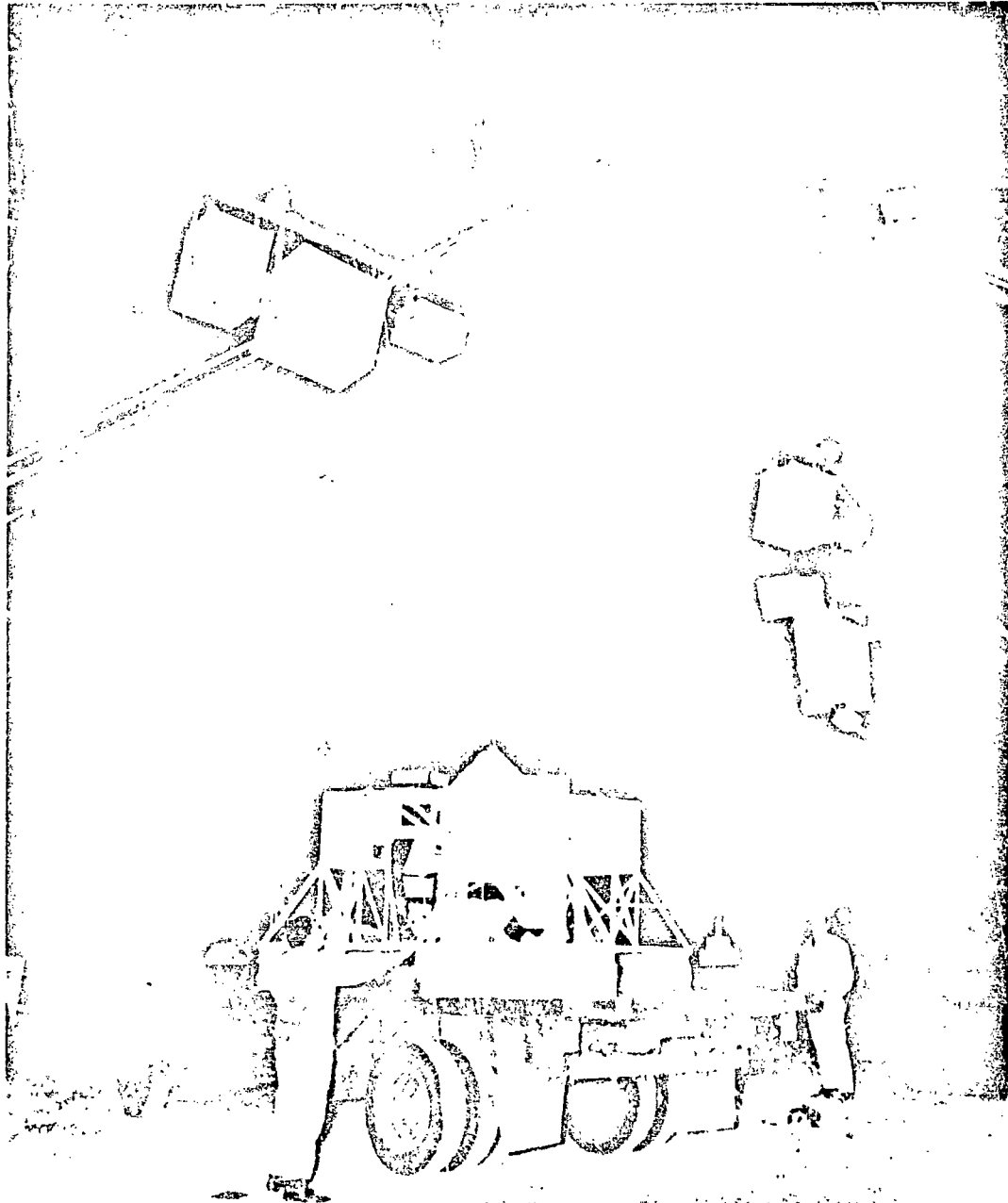GONDOLA

Figure 3-12. Flight Configuration
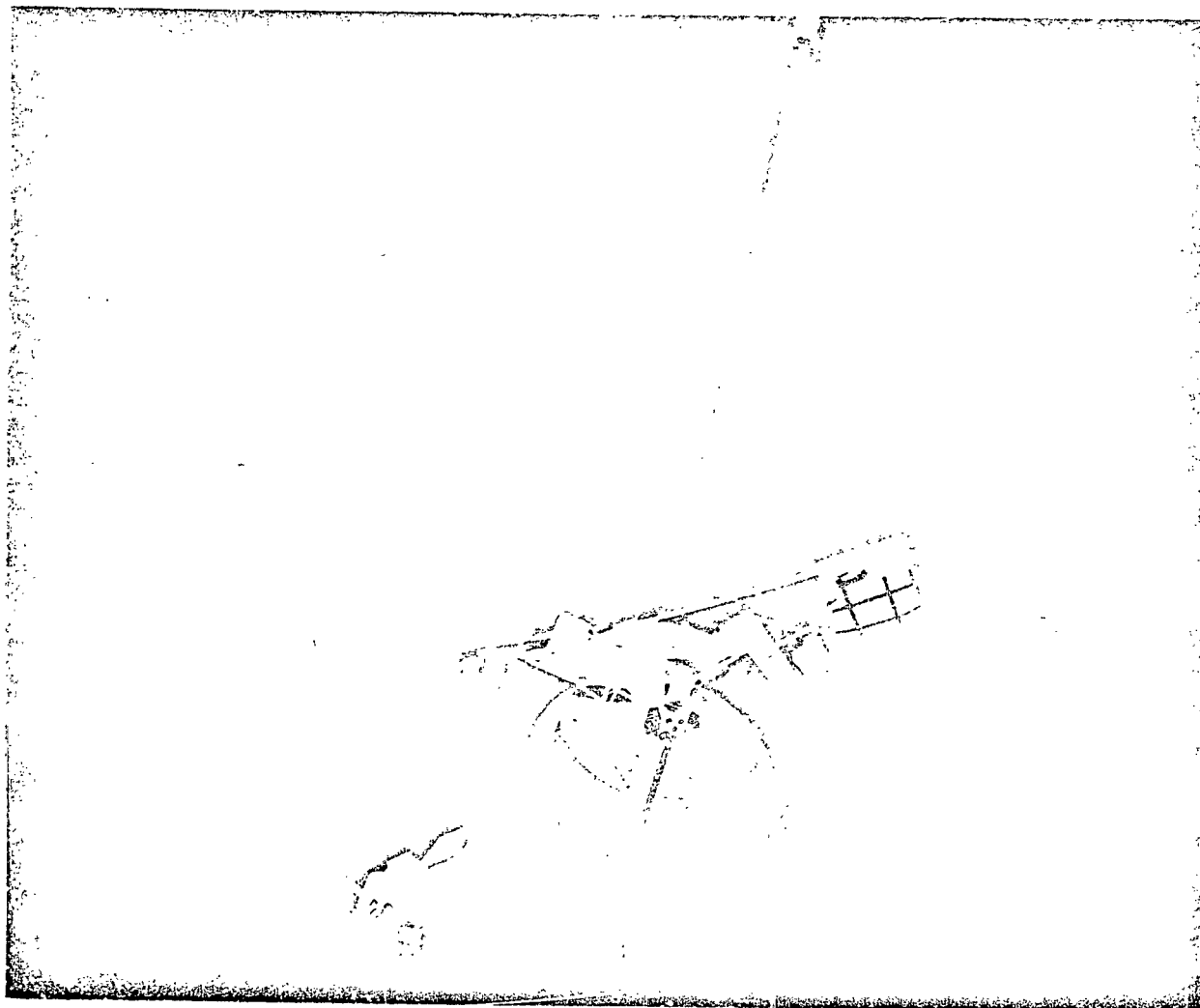
Figure 3-13. BAPE Payload Suspension

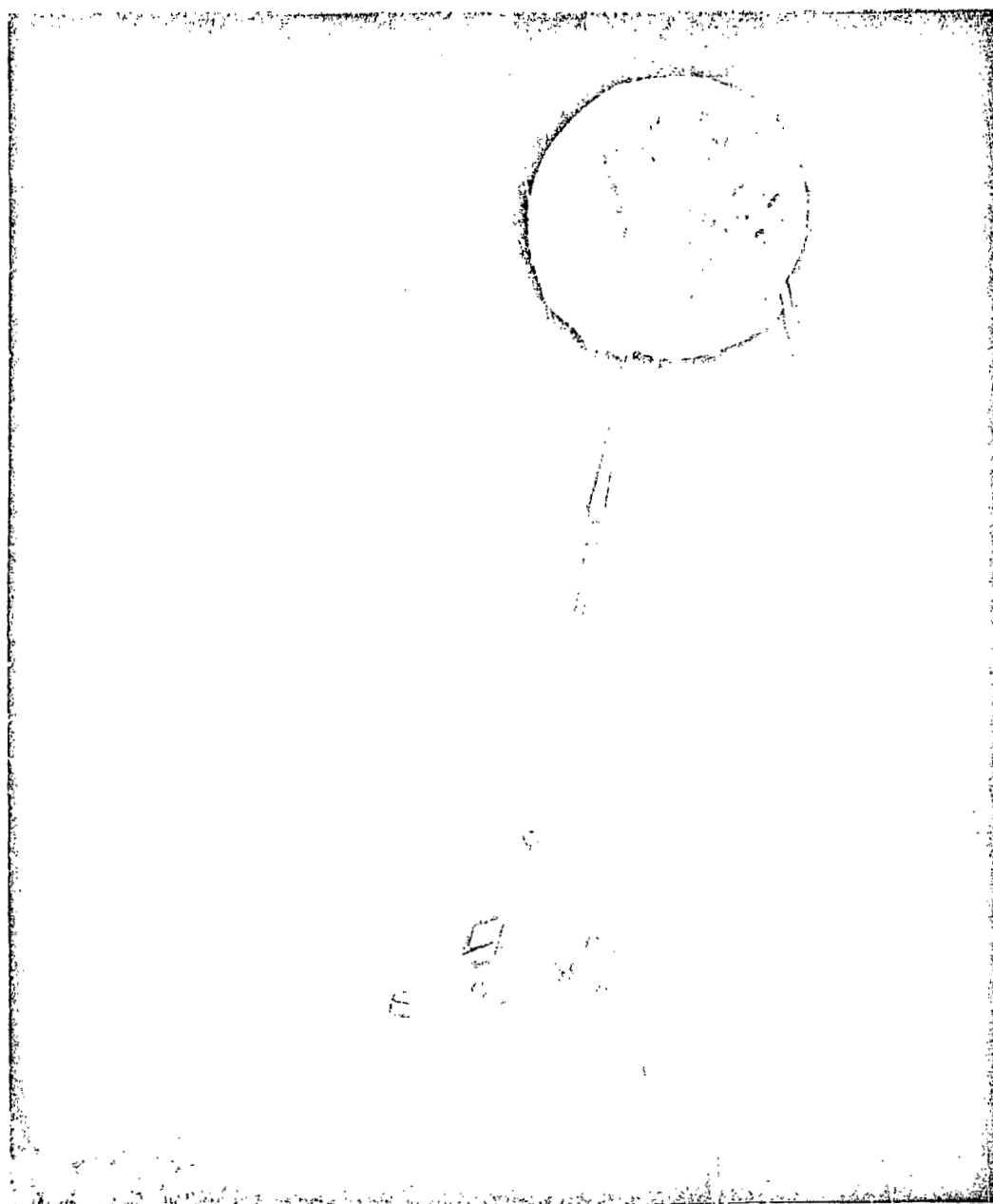Figure 3- 4.  BAPE Flight Configuration
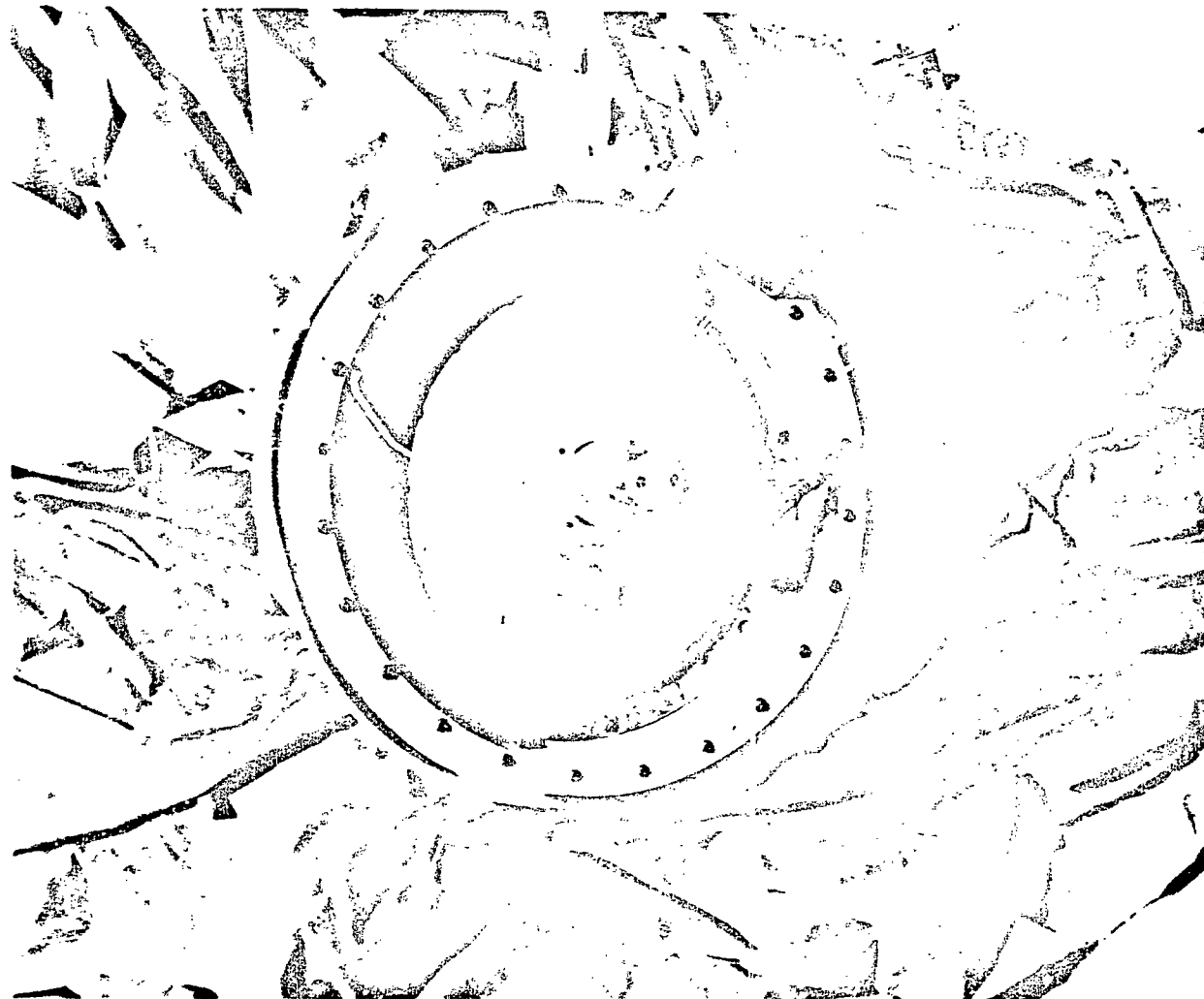
Figure 3-15. BAPE Flight Configuration

Figure 3-16.  Poppet Valve

Figure 3-17. BAPE Parachute
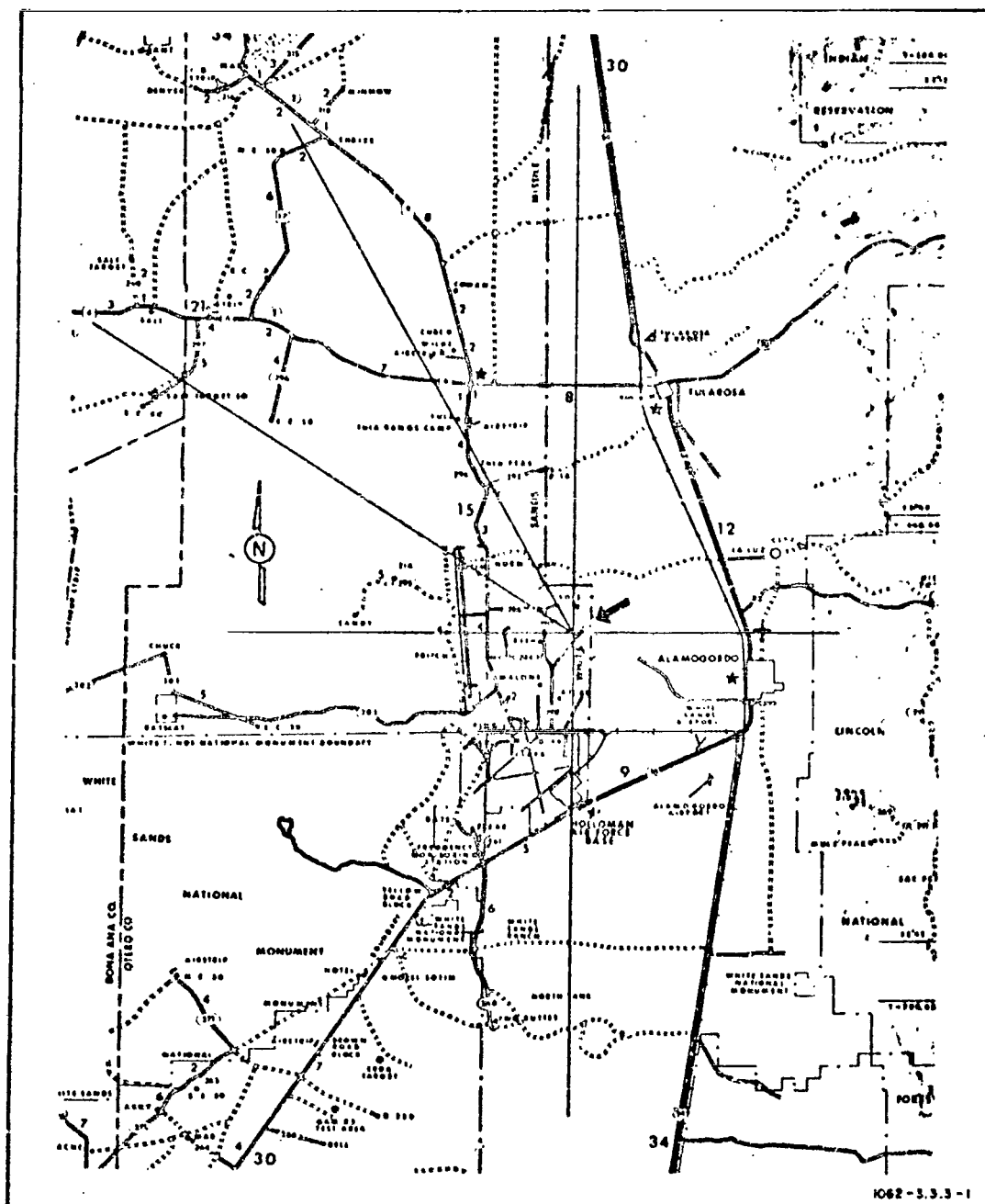
Figure 3-18. BAPE Ground Site

The balloon was launched at 0613Z (12:15 AM MST) from a point near Upham, New Mexico (50 miles west of the tracking site) and slowly drifted eastward arriving over the tracking site at about 1041Z (4:44 AM MST). Tracking commenced at about 1130Z (5:30 AM MST) and was completed at 1415Z (8:15 AM MST). The horizontal flight profile is shown in Figures 3-19 and 3-20. Altitude, zenith angle, and slant range are shown in Figures 3-21 and 3-22 and tabulated in Table 3-1.

## 3.4 GROUND TRACKING STATION

The following sections describe the mobile laser tracking station which was used to track the balloon-borne payload, with laser beams. The system consisted of three basic units which were transported to the site in three trailers (see Figure 3-23). An artists conception of the system is shown in Figure 3-24. The right-hand trailer as shown in the cutaway contains the lasers, modulators power supplies and other equipment necessary to operate the laser beams. The output is transmitted through a series of mirrors in the tracking mount towards the target. The second trailer contains the tape recorders, balloon control system telemetry, and other instrumentation necessary for the experiment. In actual operation the main optics of the telescope were used only for reception purposes, and the laser beams were transmitted from a port to the right of the main telescope. The artists conception is incorrect in this aspect. The system was developed for the BAPE experiment at GSFC Code 524.

## 3.4.1 LASER TRANSMITTER SYSTEM

The laser transmitter system, which was housed in the instrumentation trailer shown in Figure 3-23, projected its output through a tube into the tracking mount where a series of mirrors directed the radiation to the target. The transmitter system, as shown in Figure 3-25, consisted of two lasers: an RCA Model J-15268 and a Sylvania Model 948-1. Table 3-2 lists the characteristics for each of the lasers.

The lasers, which were bolted to a steel table inside the instrumentation van, remained stationary throughout the experiment. The laser beams, after being passed through collimation and alignment optics, were projected coaxially into the tracking mount. The output characteristics for the transmitter are shown in Table 3-3.
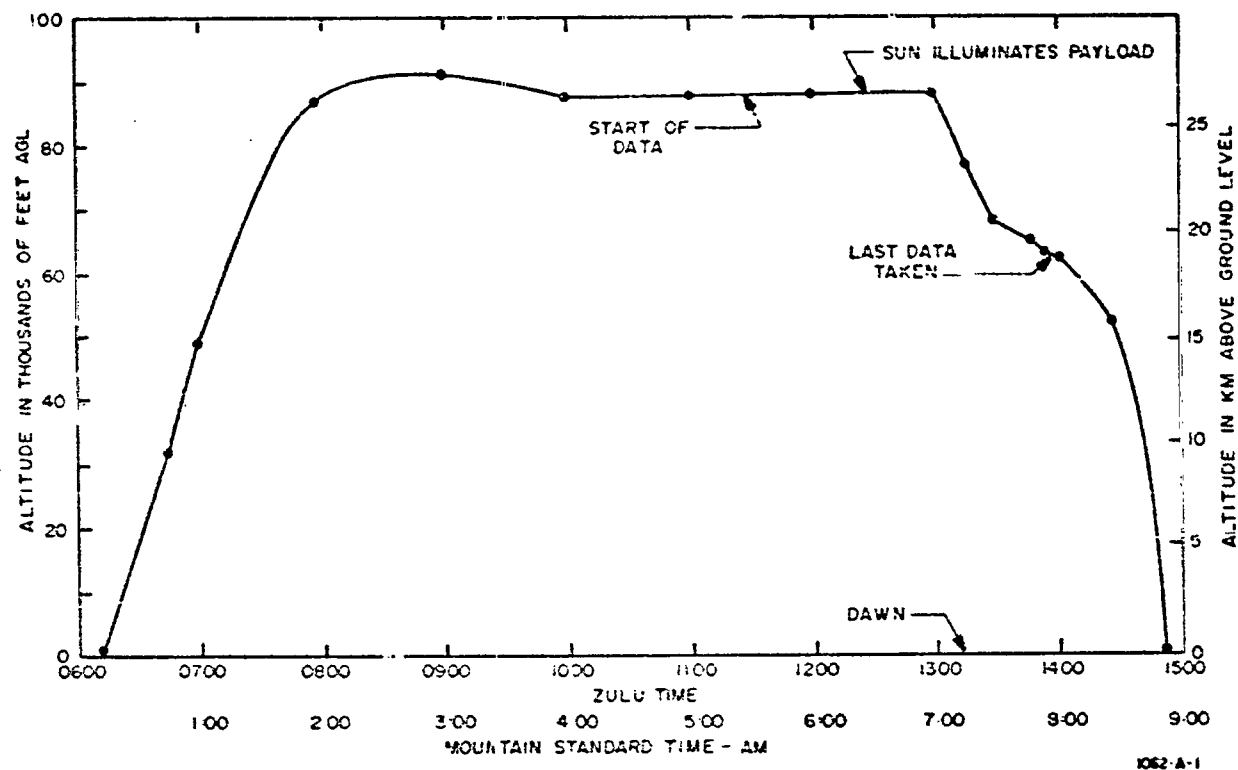
Figure 3-19. Vertical Flight Profile
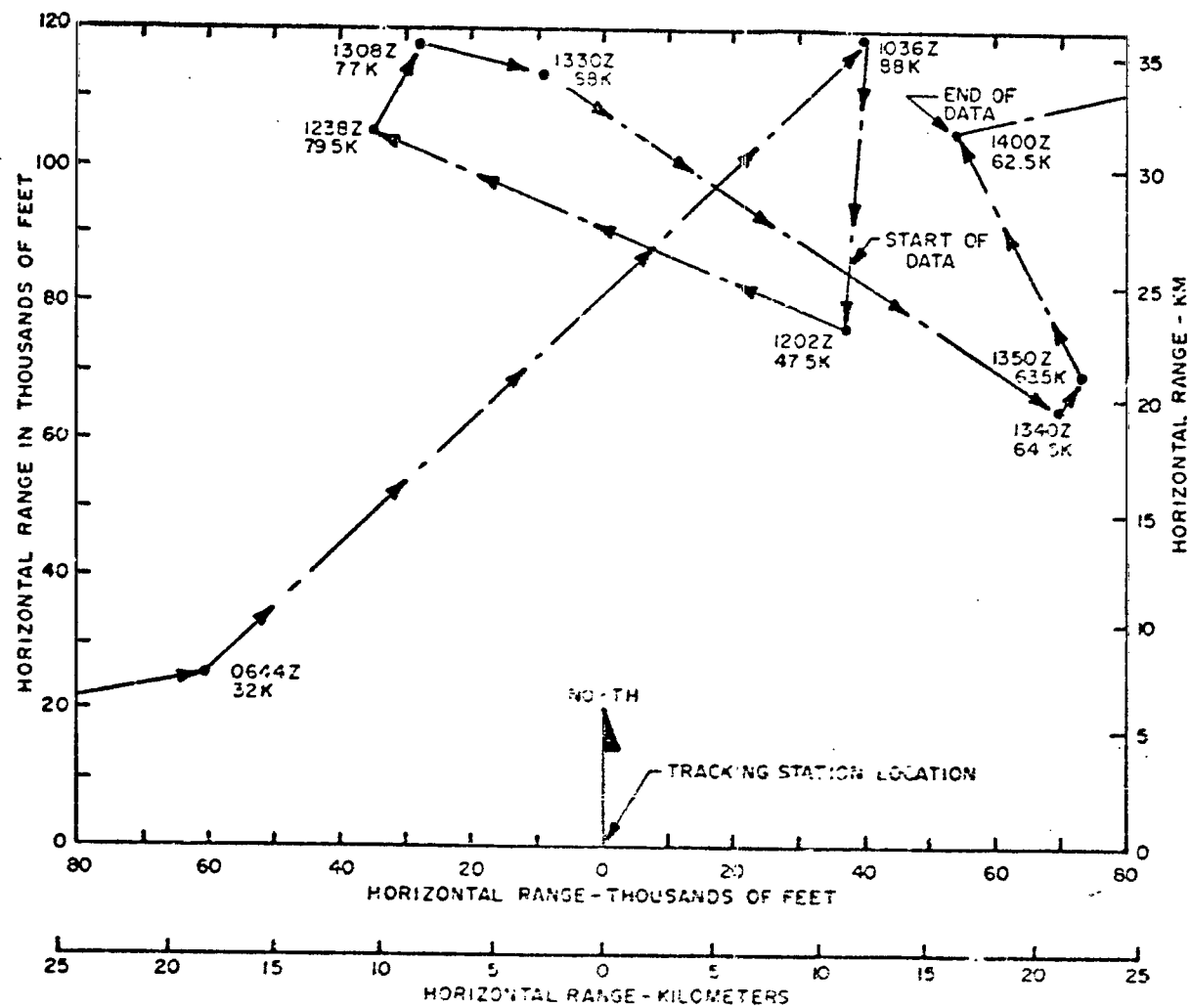
Figure 3-20. Horizontal Flight Profile
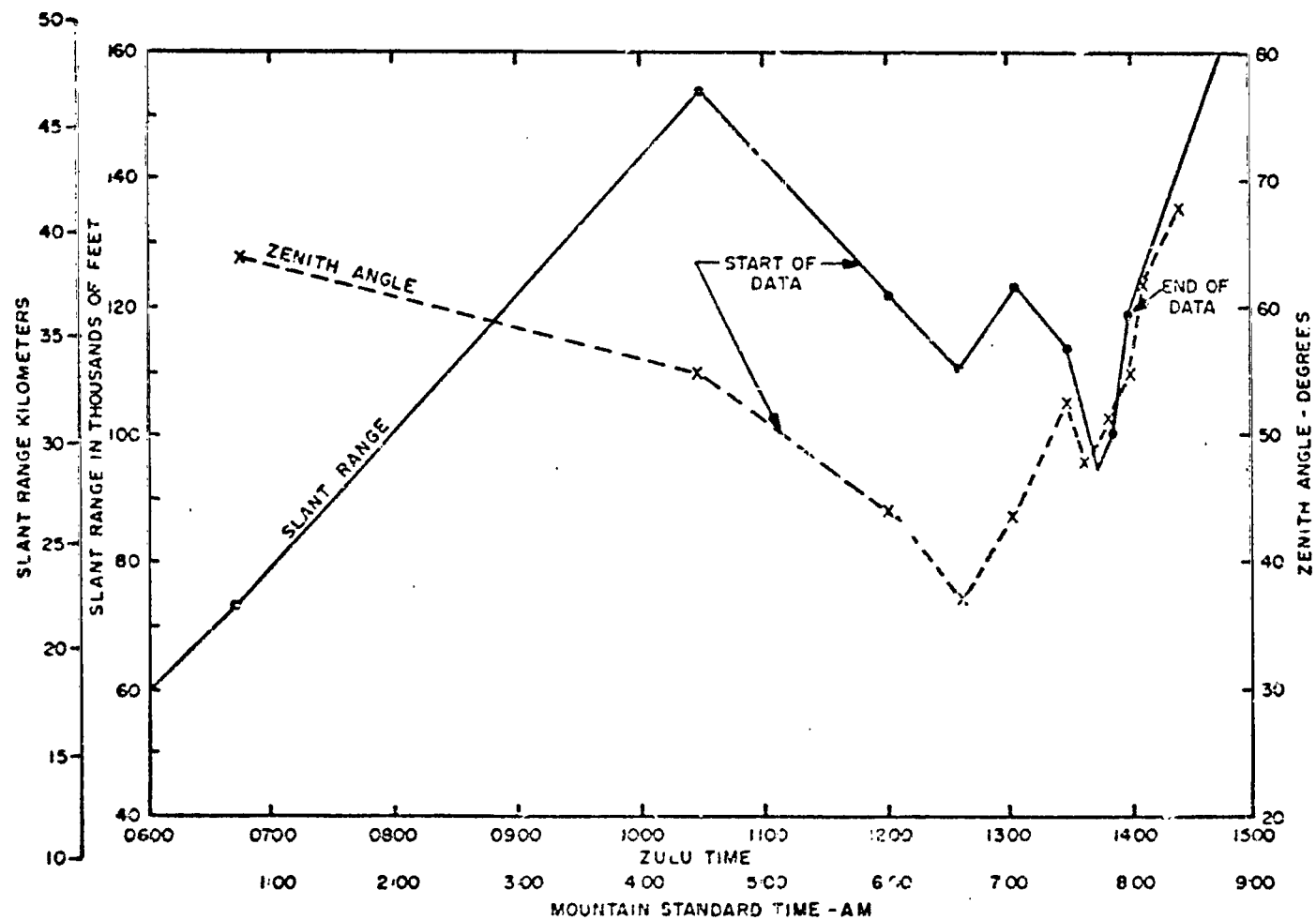
Figure 3-21. Altitude, Zenith Angle, and Slant Range

# TRAJECTORY      FLT. NO. H70-63

## DATE  21 OCT 70
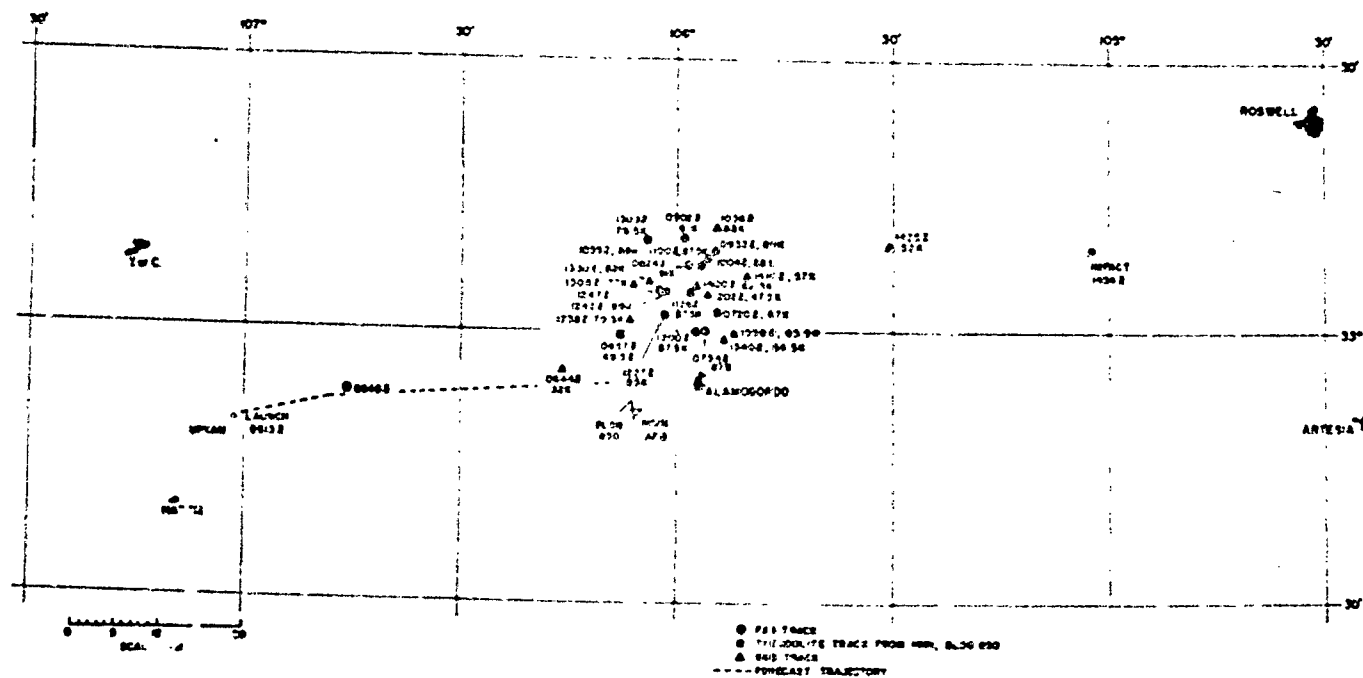


Figure 3-72.  Trajectory

1062-3.3.3-2

Table 3-1
Parameters of Balloon Trajectory, October 21, 1970

| Time (Z) | H | $R_H$ | R | $\phi$ | $\Theta$ |
|---|---|---|---|---|---|
| 0644 | 32,000 | 66,000 | 73,000 | $292^o$ | $64^o$ |
| 1034 | 88,000 | 127,000 | 154,000 | $19^o$ | $55^o$ |
| 1292 | 88,000 | 85,000 | 122,000 | $26^o$ | $44^o$ |
| 1235 | 88,000 | 67,000 | 111,000 | $342^o$ | $37^o$ |
| 1305 | 88,000 | 85,000 | 122,000 | $347^o$ | $44^o$ |
| 1330 | 68,000 | 81,000 | 114,000 | $356^o$ | $53^o$ |
| 1340 | 64,500 | 79,000 | 93,000 | $47^o$ | $47^o$ |
| 1350 | 63,500 | 79,000 | 101,000 | $47^o$ | $51^o$ |
| 1400 | 62,500 | 91,000 | 119,000 | $27^o$ | $55^o$ |
| 1410 | 57,000 | 109,000 | 12,000 | $38^o$ | $62^o$ |
| 1425 | 52,000 | 134,000 | 144,000 | $57^o$ | $68^o$ |

H - Altitude in thousands of feet

$R_H$ - Horizontal Range in thousands of feet

R - Slant Range in thousands of feet

$\phi$ - True azimuth in degrees

$\Theta$ - Zenith angle in degrees
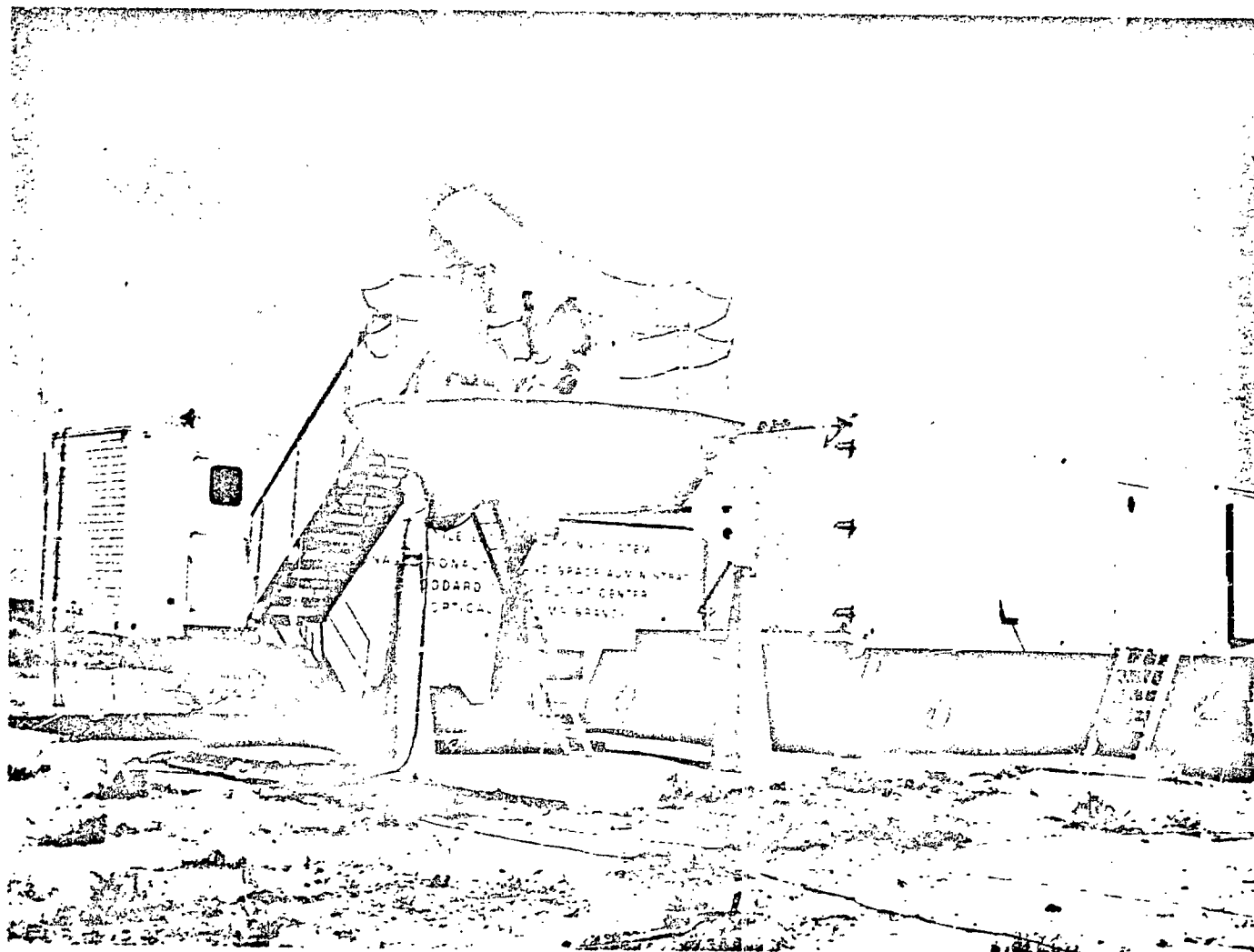
Figure 3-23. BAPE Instrumentation Trailer
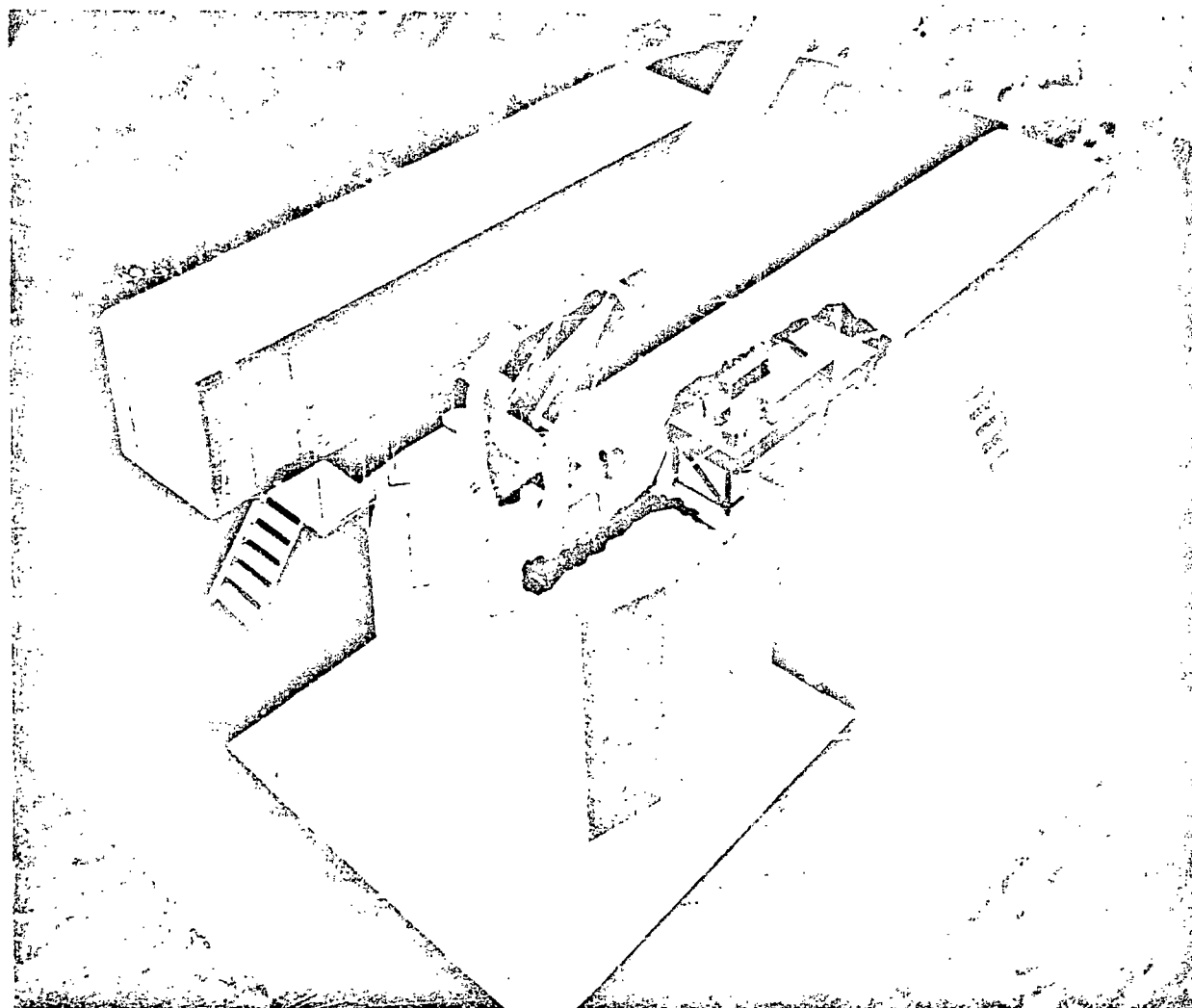
NASA G-70-3960

Figure 3-24. BAPE Mobile Tracking Station
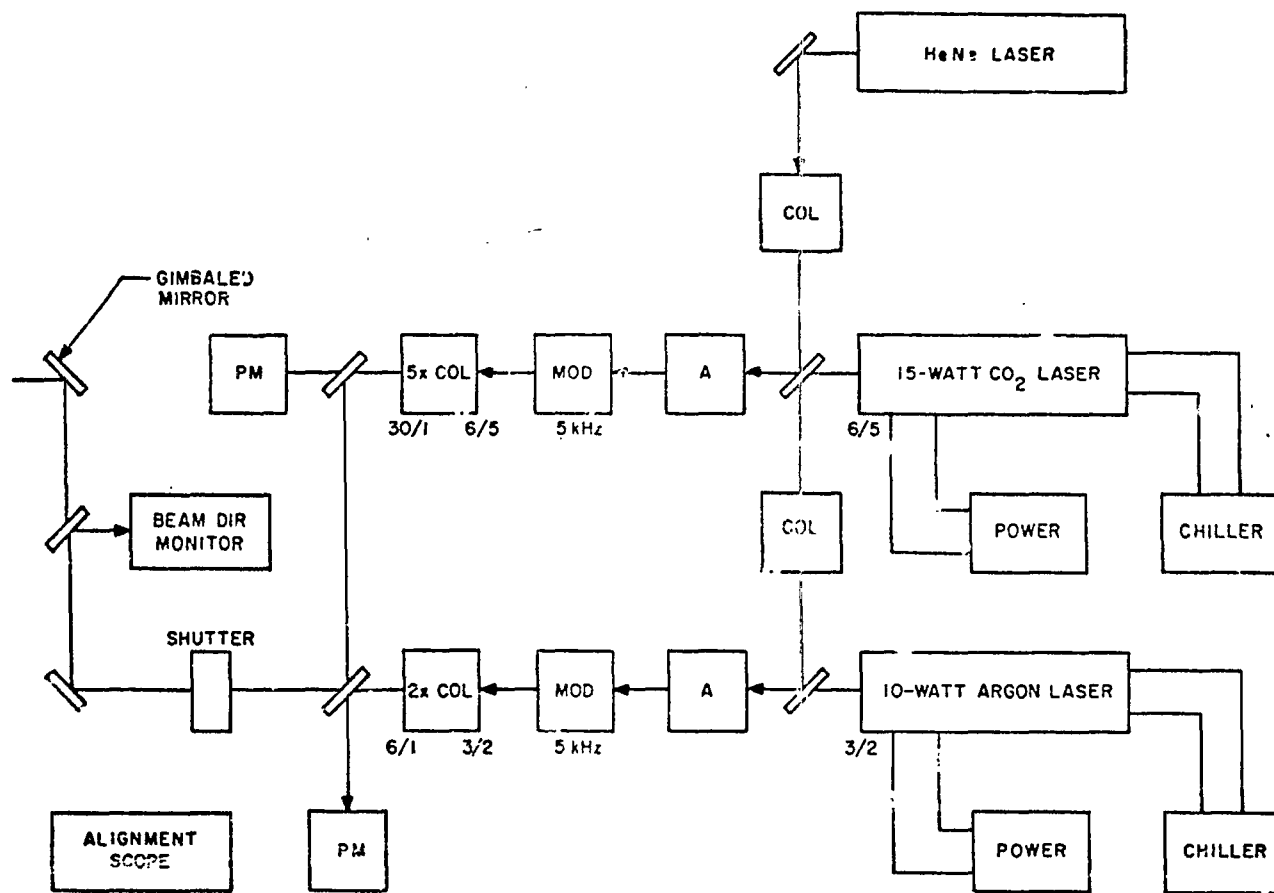
Figure 3-25. BAPE Transmitter System Block Diagram

Table 3-2
Laser Characteristics

| Characteristic | Value | |
|---|---|---|
| | RCA Model J-15268 | Sylvania Model 948-1 |
| Type | Argon ion | $CO_2$ molecular |
| Mode | CW | CW |
| Wavelength (microns) | 0.5145 | 10.6 |
| Maximum output (watts) | 4.3 | 15 |
| Beam divergence (milliradians) | 2.0 | 5 |
| Beam diameter (millimeters) | 4 | 6 |

Table 3-3
Transmitter Characteristics

| Characteristic | Value | |
|---|---|---|
| | Argon | $CO_2$ |
| Wavelength (microns) | 0.5145 | 10.6 |
| Mode (sine wave modulated) | 5 kHz | 5 kHz |
| Power out (mean watts) | 1.0 | 5 |
| (peak watts) | 2.0 | 10 |
| Divergence (milliradians) | 1.0 $(1/e^2)$ | 1.0 $(1/e^2)$ |
| Near-field diameter (millimeters) | 6 $(1/e^2)$ | 30 $(1/e^2)$ |

The $CO_2$ and argon laser beams, as shown in Figure 3-25, passed through a dichroic beam splitter, an attenuator, a 5-kHz modulator, and a collimator (5x for the $CO_2$ beam, 2x for the argon beam) to dichroic beam combining mirror. The combined beams then passed through a safety shutter to the two mirrors that aligned the beams to the tracking mount. The first of these mirrors was slightly transparent in the visible range, which allowed a target alignment scope to view a direction parallel and coincident to the outgoing laser beams. A beam direction monitor, located between the alignment mirrors, allowed alignment of the target alignment scope to the laser beams, but was removed when the system was in operation. The target alignment scope had a protective filter for the eyepiece which allowed the operator to view the target during laser operation. During tracking, the operator could view the target in the alignment scope through the coelostat of the tracking mount to ensure that the laser beams were on target. The operator adjusted the alignment scope crosshairs onto the target by means of the last alignment mirror in the transmitter system.

Power monitors, mounted in the system, monitored the output power of the lasers and the modulation frequency. Because of the difficulty and danger involved in aligning the

transmitter optical components with $CO_2$ and argon lasers, a helium-neon laser, also mounted on the transmitter table, was used as an alignment tool for the optics.

### 3.4.2 TRACKING MOUNT

The tracking mount, shown in Figure 3-26, received the radiation from the laser transmitter system and directed the radiation through a coelostat system to the target. A diagram of the coelostat system is shown in Figure 3-27. Radiation from the lasers traveled horizontally through a pipe from the laser van to a mirror at the bottom of the azimuth axis. (The pipe from the van to the tracking mount is shown in Figure 3-24.) This mirror deflected the laser beams to a second mirror at the top of the azimuth axis, which then def'ected the beams to a third mirror at the end of the elevation axis. The third mirror deflected the laser beams to a fourth mirror located slightly to the right of the main telescope. This mirror directed the laser beams through a small tube which is, as shown in Figure 3-26, to the right and parallel to the line of sight of the main telescope. The laser beams were directed through this tube without being collimated by the receiver optics.

The receiver optics, a 450-inch (11.4 m) focal length, 30-inch (76 cm) diameter cassegrain, were used to collect laser radiatio reflected from the cube corners mounted in the balloon payload, and to focus the radiation on a star tracker at the back of the telescope (see Figure 3-26). The collected radiation was used by the star tracker as a signal for automatic tracking of the target.

The tracking accuracy of the star tracker was approximately 5 arc seconds rms; however, because of intermittent loss of autotrack due to severe scintillation during the experiment, the target had to be manually tracked by the observer (shown in Figure 3-28). Manual tracking was accomplished by observing the target through a guide telescope and adjusting the main telescope by means of a joy-stick control mounted on the manual control panel. Details for the guide scope and control panel are shown in Figures 3-26 and 3-28. Tracking accuracy in the manual mode was approximately 30 arc seconds rms.

In the autotrack mode, the star tracker intermittently lost track and went into the search mode because of insufficient integration time in the star tracker circuitry. After this experiment, modifications were made to correct this problem with the star tracker circuitry, and the star tracker tracked perfectly for BAPE II flights during September of 1971.

1062-3.4.3-1

Figure 3-26. BAPE Tracking Mount

#3 COUDE FLAT

SECONDARY

EYEPIECE

#4 COUDE FLAT
(IMAGE MOTION
COMPENSATION
AT EXIT PUPIL)

76 CM (30 INCH)
PRIMARY

#5 COUDE FLAT

#6 COUDE FLAT

LASER
INSTRUMENTATION
VAN

VIBRATION
ISOLATED
TABLE

AZ/EL TRACKING
PEDESTAL
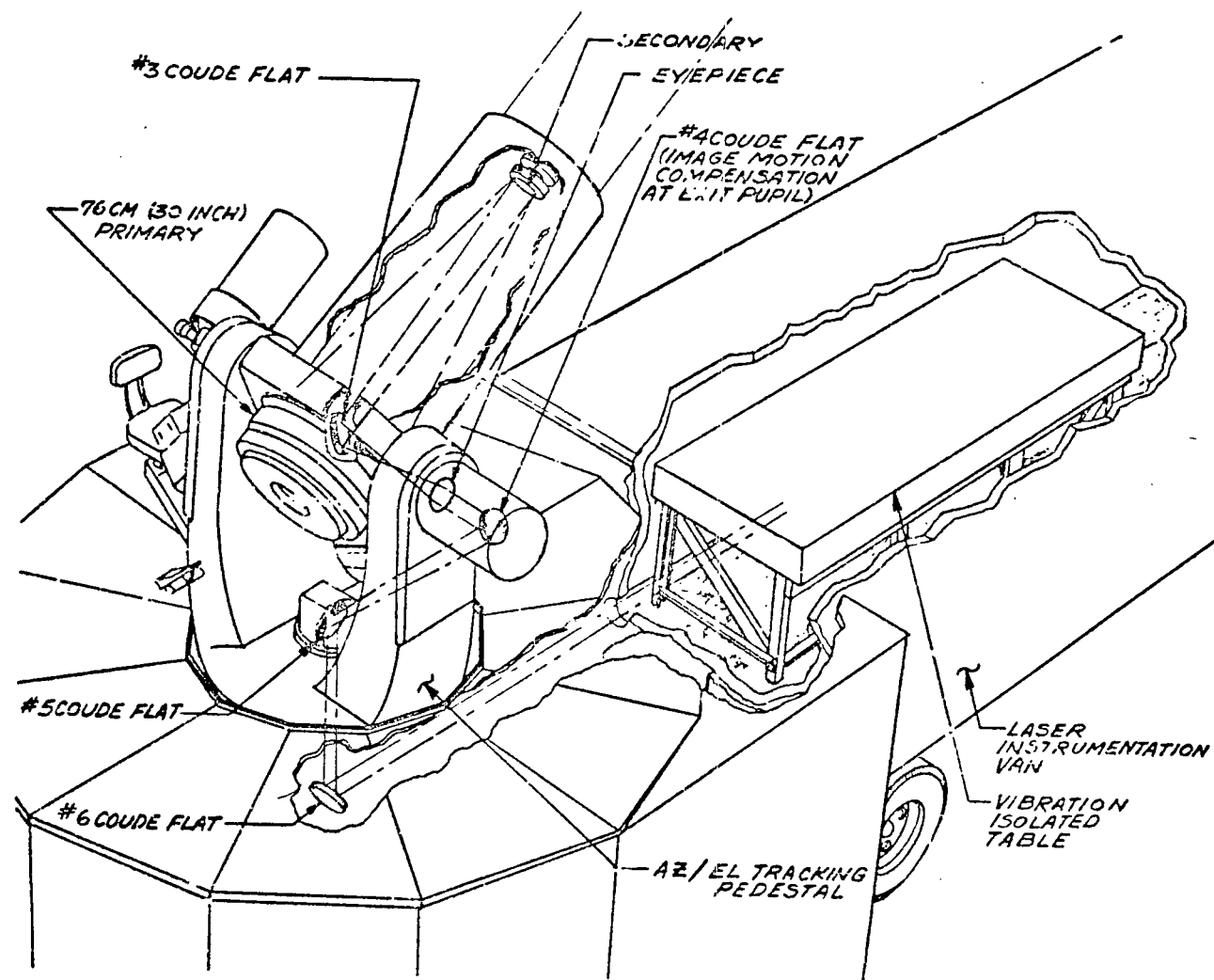
Figure 3-27. Coelostat System Diagram
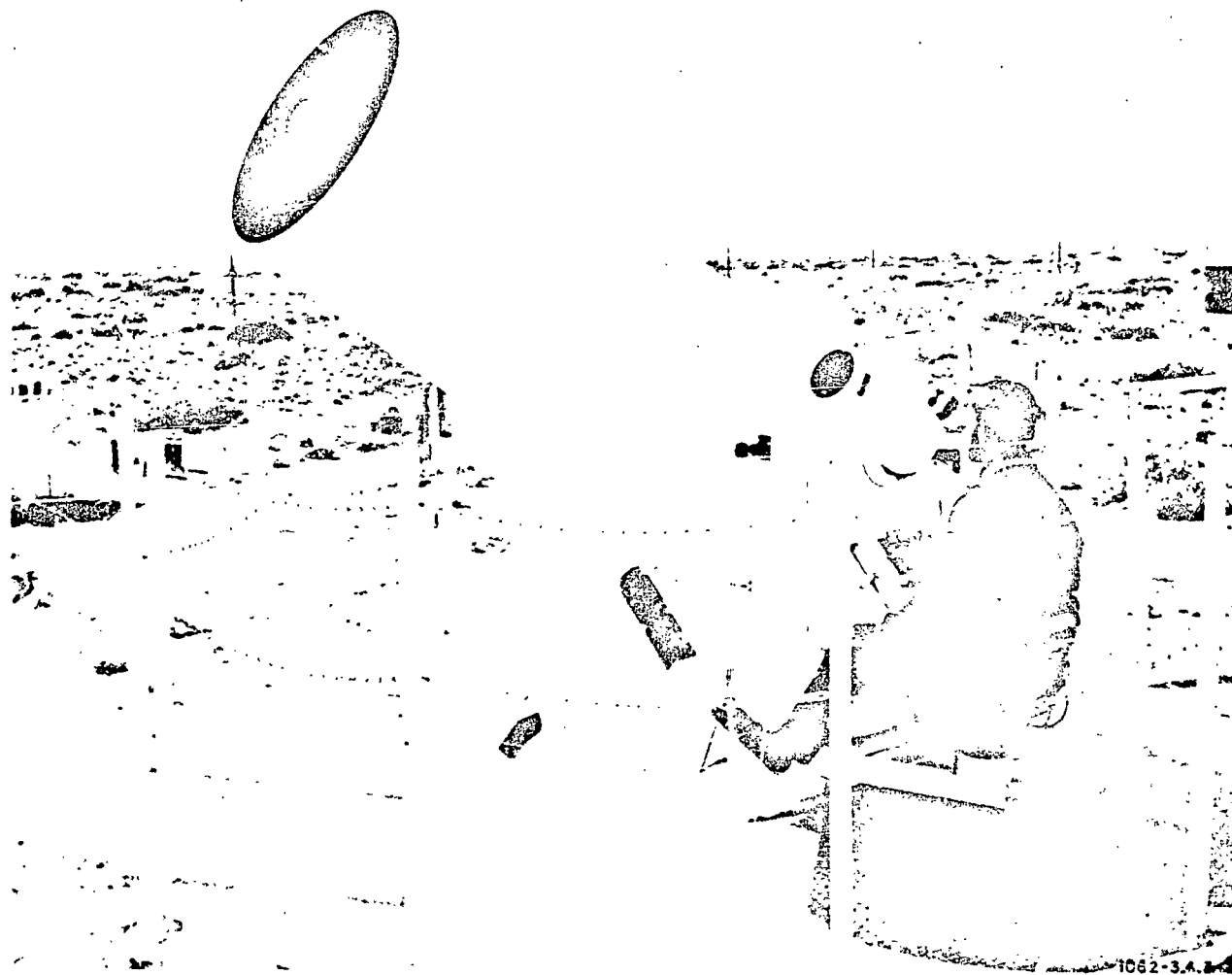
Figure 3-28. BAPE Tracking Mount

The tracking mount is a converted NIKE-AJAX tracking pedestal to which the 30-inch telescope and coelostat system have been added. During operation, the tracking mount is elevated by the aluminum legs shown in Figure 3-24 so that the elevation axis is 15 feet (5 meters) above ground level. The tracking mount is elevated for the following reasons:

- To give a clear view of the horizon over trailers and buildings in the surrounding area

- To provide a safety factor by keeping the laser beams well above personnel in the area

- To keep the laser beams above the low level ground turbulence which does not follow the normal Kolmogorov spectrum of turbulence

## 3.4.3 TELEMETRY, PAYLOAD COMMAND, AND DATA RECORDING SYSTEM

Command of the balloon payload and data from the balloon payload were obtained through an L- and S-band telemetry system. A console (Figure 3-29) in the instrumentation van allowed an operator on the ground to control the azimuth, elevation, and operational status of the payload. Two small parabolic telemetry antennas (Figure 3-30) transmitted the information to and received information from the payload. In addition to the command and telemetry system for the payload, a standard radiosonde transmitter operating between 1660 and 1700 MHz was used to transmit data from the thermosonde to the ground. Telemetry characteristics are listed in Table 3-1.

Table 3-1
Experiment Payload Command and Telemetry

| Characteristic | Value |
|---|---|
| 1431-MHz command system | Conic Corp. Model CTM-UHF-305 |
| Output | 5 watts |
| Antenna | 4-foot paraoblic, 10-inch primary beamwidth |
| | |
| 2241.5-MHz telemetry | Conic Corp. Model CTM-UHF-305 |
| Output | 5 watts |
| Antenna | Hemispherical pattern, omnidirectional |
| | |
| 1660- to 1700-MHz telemetry | Viz Mfg. Model 1091, military Model No. AN/AMT-12A |
| Output | 400 milliwatts |
| Bandwidth | 500 Hz |
| Modulation | 100%, audio |
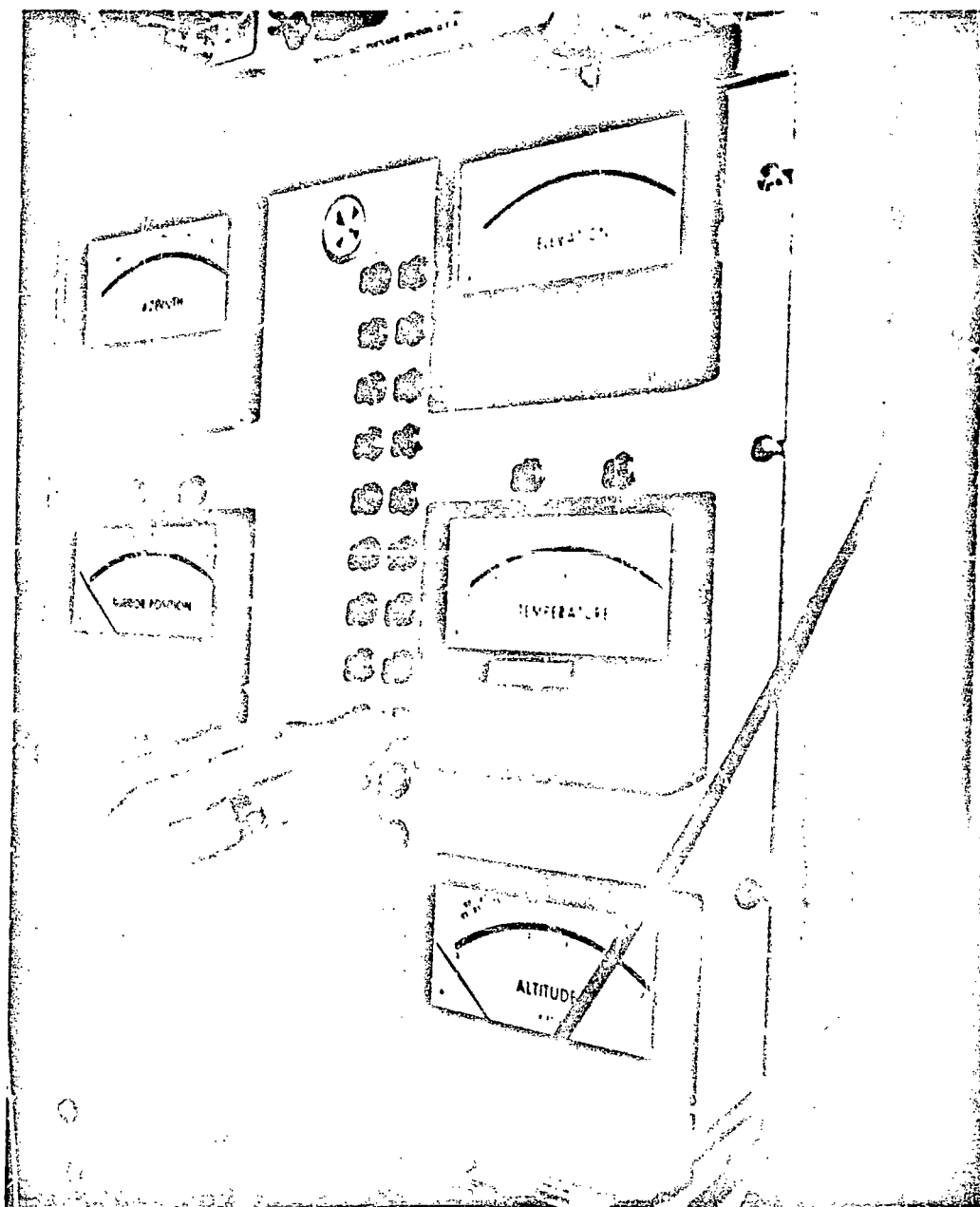
Figure 3-29. Control Console
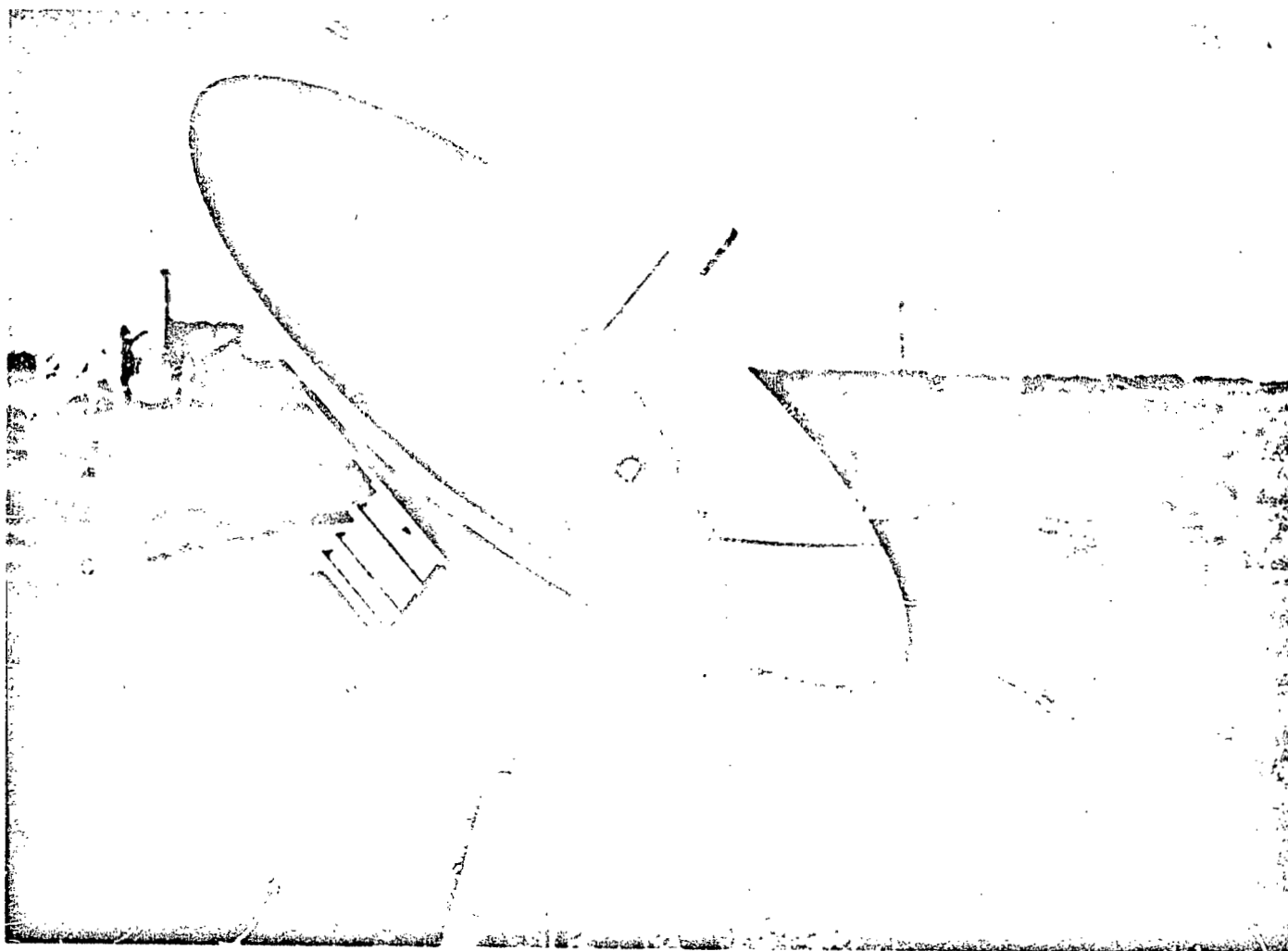
Figure 3-30. Telemetry Antenna

Downlink telemetry carrier assignments are shown in Table 3-5. The outputs of these channels were recorded on an FM tape recorder. Data recording assignments and recording circuits are shown in Table 3-6 and Figure 3-31, respectively.

Table 3-5
Downlink Telemetry Carrier Assignments

| Channel | Function | Bandwidth |
|---------|----------|-----------|
| 3B | Spare | |
| 5B | Spare | |
| 11B | Argon movable | |
| 13B | $CO_2$ fixed | 1-kHz constant bandwidth |
| 7B | Spare | |
| 9B | Argon fixed | |
| 15B | $CO_2$ movable | |
| 4 | Tracker acquisition indicator | |
| 5 | Pressure | |
| 6 | Balloon receiver AGC | |
| 7 | Commutated temperatures | |
| 8 | Mirror position | Proportional bandwidth |
| 9 | Azimuth | |
| 10 | Elevation | |
| 11 | Spare | |
| 12 | Azimuth error | |
| 13 | Elevation error | |

Table 3-6
Data Recording Channels

| T.R. Channel | Signal | Bandwidth | Record Module | | |
|---|---|---|---|---|---|
| #1 | Argon A fixed | 1 kHz | FM | | |
| #2 | Argon B movable | 1 kHz | FM | | |
| #3 | $CO_2$ A fixed | 1 kHz | FM | | |
| #4 | $CO_2$ B movable | 1 kHz | FM | | |
| #5 | 36-bit BCD time (1-kHz carrier) | | Direct | | |
| #6 | Balloon pack az error | DC | FM | | |
| #7 | Balloon pack el error | DC | FM | | |
| #8 | Star track az error | DC | FM | | |
| #9 | Star track el error | DC | FM | | |
| #10 | Ground rec argon demod | 1 kHz | FM | | |
| #11 | Ground rec $CO_2$ demod | 1 kHz | FM | | |
| #12 | Ground comm data | DC | FM | 4 | 1. Track acq ind |
| #13 | Trans power out (argon) | (5 kHz sin) | Direct | 5 | 2. Press |
| #14 | Trans power out ($CO_2$) | DC | FM | 6 | 3. Ball rec AGC |
| | | | | 8 | 4. Mirror position |
| #1 | BAPE commutated temp | DC | FM | 9 | 5. Balloon pack.az |
| #2 | 36-bit BCD | (1-kHz carrier) | Direct | 10 | 6. Balloon pack.el |
| #3 | Grd thermal sensor sig | 1 kHz | FM | M | 7. N/A position az |
| #4 | Grd thermal sensor sig | 1 kHz | FM | M | 8. N/A position el |
| #5 | Wind velocity | 1 kHz | FM | | 9. Calib |
| #6 | rec argon | 5 kHz ± 1 kHz | Direct | | 10. |
| #7 | Grd rec $CO_2$ | 5 kHz ± 1 kHz | Direct | | |
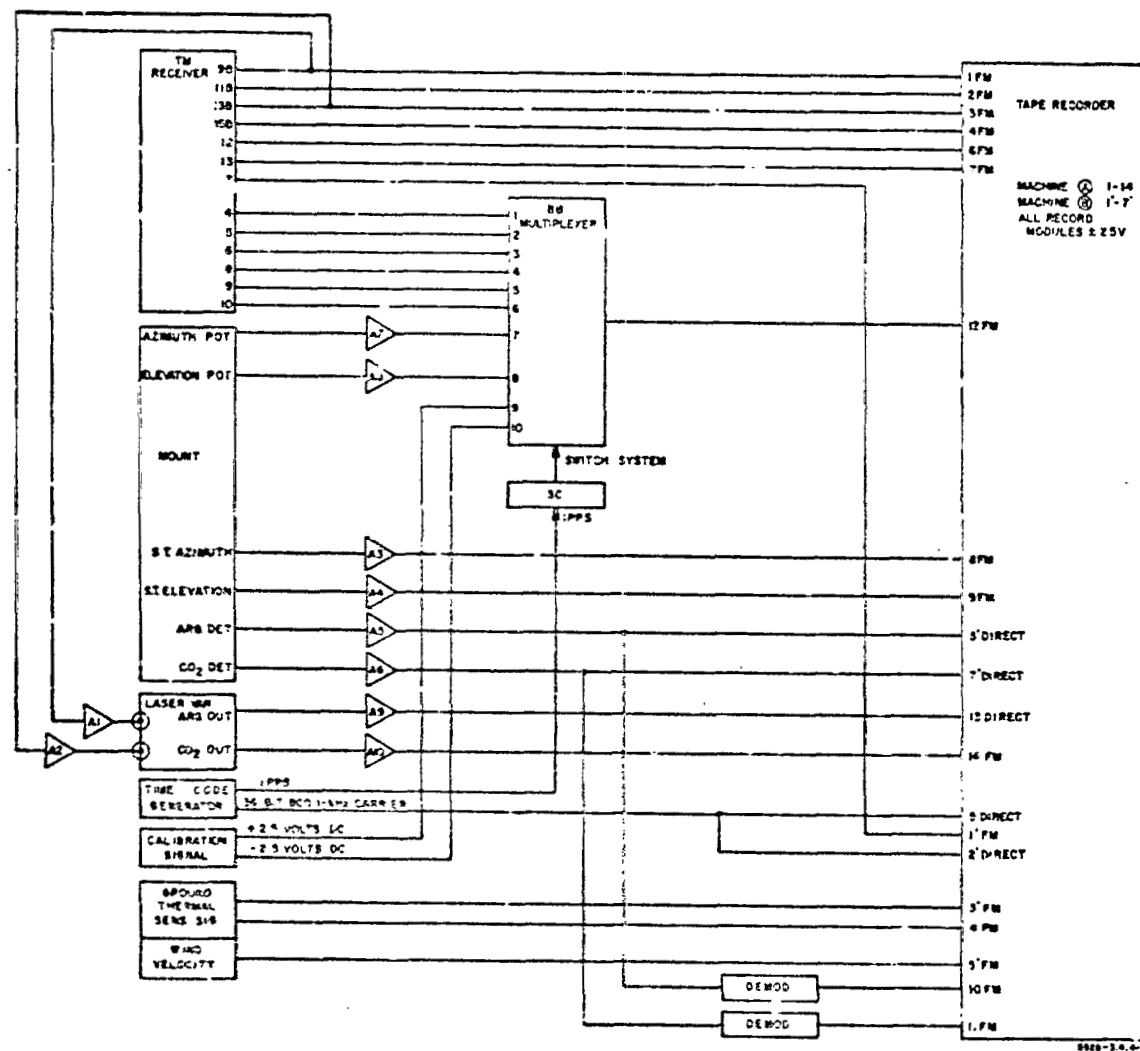
Figure 3-31. Data Recording Assignments and Circuits

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14. Calibration will account for whatever non-uniformities remain.

15. Ed. Charles D. Dodgman, Handbook of Chemistry and Physics, Chemical Rubber Pub. Co., Cleveland, Ohio (1956), pp. 2364, 2368.

16. This circuit is the result of a collaboration with Professor J. Richard Kerr of the Oregon Graduate Center.

## 4. CHRONOLOGY OF EXPERIMENT

This section explains the time sequence of events which occurred during the experiments and gives a chronological description of the events.

The balloon was launched at 12:13 AM MST from Upham, New Mexico, a point nearly 50 miles west of the tracking site. The balloon ascent rate was set at approximately 1000 feet per minute. At approximately 12:15 AM MST when the balloon had risen to an altitude of 1000 feet, the reel supporting the thermosonde was released and about 20 seconds later the sonde was fully deployed 1000 feet below the main payload. At 12:30 AM MST the flashing strobe lights of the balloon were spotted from the tracking station and from that time on until termination of the experiment the tracking station was in visual contact with the balloon. The weather was clear with no clouds in sight during the experiment. High winds in the 40,000 to 80,000 ft altitude range drove the balloon quickly eastward at speeds of up to 90 knots, and by 1:00 AM MST the payload was close enough to command by telemetry. A check of payload operation was made and everything was found to be in order. During ascent, a radiosonde receiver at the launch site was able to receive data from the thermosonde to an altitude of about 20,000 feet. A second receiver, located at the tracking site with a higher gain antenna was supposed to pick up the data collection at this point but due to a malfunction was unable to operate. A baroswitch on the thermosonde automatically shut it off at 45,000 feet. After reaching a float altitude of about 92,000 feet, the balloon was slowly manuevered towards the tracking station by slightly changing altitude to allow the prevailing winds to drive the balloon towards the tracking station. Because of the low velocity of the winds, the balloon was not within tracking range until about 5:30 AM. At 5:37 AM tracking and data taking was started and continued until 6:10 AM, at which time, data taking was stopped in order to make a quick analysis of the data to make sure that all systems were working correctly. At 7:05 AM tracking was resumed and continued until the balloon went out of range at 8:00 AM. Dawn occurred at 7:13 AM, and therefore most of the data was taken during the dawn calm, the period of lowest turbulence. At about 7:00 AM a slow descent of the balloon was started as shown in paragraph 3.3.3 and at 8:30 AM the flight was terminated. The payload descended by parachute from an altitude of 45,000 feet and landed near Roswell, New Mexico.

## 5. DATA REDUCTION

The following sections discuss the method of data reduction used in reducing the experiment data from raw form to final results.

### 5.1 THERMOSONDE DATA

Table 5-1 lif+ : th· strength of turbulence versus height as obtained from the thermosonde. Figure 5-1 shows the product of strength of turbulence, and height to the 5/6 power versus height. This product controls the strength of scintillation and therefore the figure shows that the bulk of the scintillation caused by turbulence below 6 km was caused by a layer at approximately 700 meters above the ground.
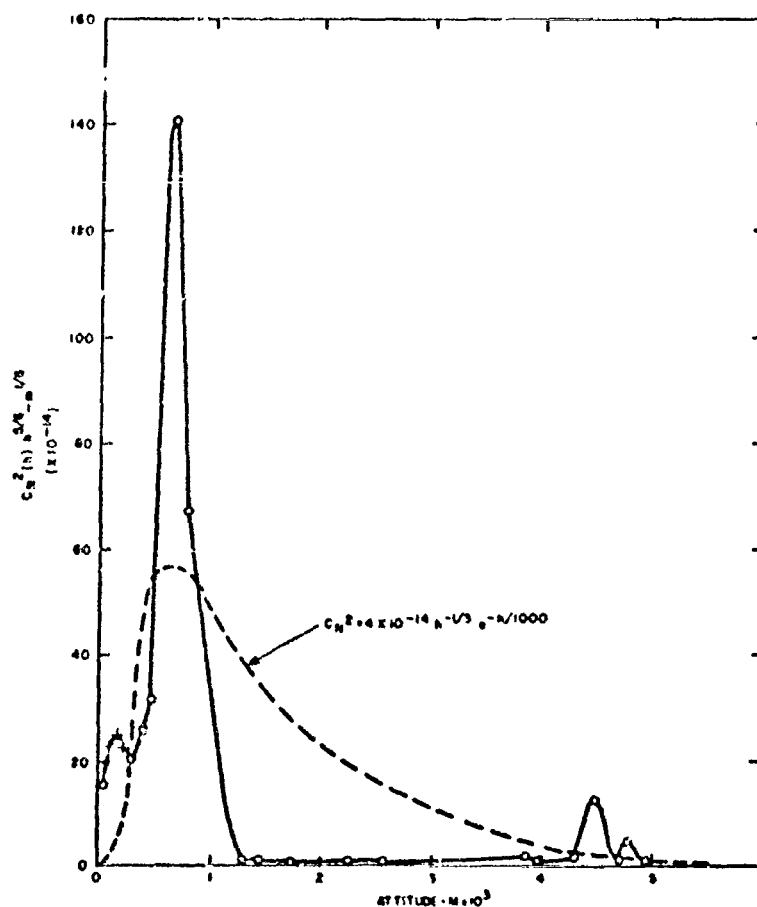


Figure 5-1

Table 5-1
Thermosonde Data

| h | (h-hg) | $(h-hg)^{5/6}$ | $(h-hg)^{5/3}$ | $C_N^2$ | $C_N^2(h)\,(h-hg)^{5/6}$ | $C_N^2(h)\,(h-hg)^{5/3}$ |
|---|---|---|---|---|---|---|
| | | | | Units | | |
| m | m | $m^{5/6}$ | $m^{5/3}$ | $m^{-2/3}$ | $m^{1/6}$ | m |
| 1490 | 290 | 113 | $1.28 \times 10^4$ | $0.18 \times 10^{-14}$ | $20.3 \times 10^{-14}$ | $2.30 \times 10^{-11}$ |
| 1560 | 360 | 134 | $1.80 \times 10^4$ | $0.19 \times 10^{-14}$ | $25.6 \times 10^{-14}$ | $3.4. \times 10^{-11}$ |
| 1650 | 450 | 164 | $2.69 \times 10^4$ | $0.19 \times 10^{-14}$ | $31.2 \times 10^{-14}$ | $5.11 \times 10^{-11}$ |
| 1850 | 650 | 210 | $4.84 \times 10^4$ | $0.64 \times 10^{-14}$ | $141 \times 10^{-14}$ | $3.10 \times 10^{-10}$ |
| 1980 | 780 | 260 | $5.76 \times 10^4$ | $0.26 \times 10^{-14}$ | $67.8 \times 10^{-14}$ | $1.76 \times 10^{-10}$ |
| 2470 | 1270 | 380 | $1.44 \times 10^5$ | $0.39 \times 10^{-16}$ | $1.48 \times 10^{-14}$ | $5.63 \times 10^{-12}$ |
| 2590 | 1390 | 410 | $1.68 \times 10^5$ | $0.24 \times 10^{-16}$ | $0.98 \times 10^{-14}$ | $4.03 \times 10^{-12}$ |
| 2900 | 1700 | 500 | $2.50 \times 10^5$ | $0.18 \times 10^{-16}$ | $0.90 \times 10^{-14}$ | $4.50 \times 10^{-12}$ |
| 3400 | 2200 | 600 | $3.60 \times 10^5$ | $0.12 \times 10^{-16}$ | $0.72 \times 10^{-14}$ | $4.32 \times 10^{-12}$ |
| 3740 | 2540 | 680 | $4.62 \times 10^5$ | $0.15 \times 10^{-16}$ | $1.02 \times 10^{-14}$ | $6.9 \times 10^{-12}$ |
| 5070 | 3870 | 980 | $9.60 \times 10^5$ | $0.21 \times 10^{-16}$ | $2.06 \times 10^{-14}$ | $2.02 \times 10^{-11}$ |
| 5145 | 3945 | 1000 | $1.00 \times 10^6$ | $0.17 \times 10^{-16}$ | $1.70 \times 10^{-14}$ | $1.70 \times 10^{-11}$ |
| 5455 | 4285 | 1050 | $1.12 \times 10^6$ | $0.10 \times 10^{-16}$ | $1.06 \times 10^{-14}$ | $1.12 \times 10^{-11}$ |
| 5670 | 4470 | 1100 | $1.21 \times 10^6$ | $1.2 \times 10^{-16}$ | $13.2 \times 10^{-14}$ | $1.45 \times 10^{-10}$ |
| 5875 | 4675 | 1130 | $1.28 \times 10^6$ | $0.079 \times 10^{-16}$ | $0.80 \times 10^{-14}$ | $1.01 \times 10^{-11}$ |
| 5950 | 4750 | 1140 | $1.30 \times 10^6$ | $0.41 \times 10^{-16}$ | $4.66 \times 10^{-14}$ | $5.33 \times 10^{-11}$ |
| 6090 | 4890 | 1190 | $1.42 \times 10^6$ | $0.097 \times 10^{-16}$ | $1.15 \times 10^{-14}$ | $1.37 \times 10^{-11}$ |

## 5.2 METHOD OF OPTICAL DATA REDUCTION

Data reduction for the two channels of argon detector data and the two channels of $CO_2$ detector data yielded the important statistics of probability density function, cumulative density function, variance, autocorrelation, power spectral density, and cross-correlation. With the exception of power spectral density, all statistics were obtained by playing back the tape-recorded analog data into a Hewlett Packard Model 3721A correlator. This instrument is a digital signal analyzer which derives the desired statistics by sampling the input data and operating on the digital result with hardwired statistical functions. All statistics were calculated in the summation mode in which N regularly spaced samples of data are used to compute the desired statistic and the result is divided by N. The sampling rate was set at $10^3$ sec$^{-1}$ for the argon detector data and $10^2$ sec$^{-1}$ for the $CO_2$ detector data. These were chosen from estimates of signal bandwidths to be small enough so that no aliasing errors were present. The value of N was set at $3.2 \times 10^4$ or $6.4 \times 10^4$ which provides, for example, a statistical average over 32 or 64 seconds of data when a 0.001-second sampling interval is used. The resultant statistic was displayed as an X-Y plot on a CRT display or hard-copied on a standard X-Y ink recorder. The output display had a resolution of 100 points equally spaced across the horizontal axis.

Resolution was 200 levels for the full vertical axis and was complemented by interpolation between points on the display.

The first processing of the data was done in the autocorrelation mode. First, the dc level of the input analog data was removed and then the autocorrelation was computed. Argon detector data analysis employed the ac coupling capability of the correlator with its 1-Hz cutoff frequency. $CO_2$ detector data was first passed through an external high-pass filter with a 0.1 cutoff frequency and then into the dc-coupled correlator. This was done to preserve more of the typically low frequency $CO_2$ detector data.

The value of the autocorrelation function at time lag zero (mean square value) was read from the CRT display and written down as a function of detector number. Simultaneous data segments were analyzed for each pair of detectors. Then the cross-correlation between like detectors was computed and its value at time lag zero recorded. This value divided by the square root of the product of both individual mean square values from the autocorrelation analysis was the desired correlation coefficient. All scale factors between input irradiance at the detector and tape recorder output voltage

cancelled out when the coefficient was formed. The correlation coefficient was computed for several detector separations and the correlation function for each wavelength was determined.

The mean square values obtained above were just the data variances since the dc or average value was zero in each case. The variances were in terms of tape recorder output voltage squared. It was desirable to normalize the variances by the appropriate dc values squared in order to have a standard summary statistic for the strength of irradiance fluctuations. The dc value for each data record was obtained from the 50% level of the cumulative density function as the data was once again played back into the correlator (this time, dc coupled). It was also necessary to compute the dc value of the background (signal from detector in absence of laser irradiance) and subtract this from the data dc value before the final ratio was computed. Several cumulative density functions were also plotted on the X-Y recorder and then transferred to probability paper to test for log-normality.

The autocorrelation function was transferred to the X-Y recorder in the same manner as above and the 100 function points were fed into a digital computer. The computer was programmed to perform a Fourier transformation of each input autocorrelation function. The result was the desired power spectral density function.

Definitions of the terms used in reducing the optical data are listed below:

$N$ — Sample time in seconds

$\rho$ — Separation of sensors - cm

$R_1(0)$ — $\overline{(S_1 - \bar{S}_1)^2}$

$R_2(0)$ — $\overline{(S_2 - \bar{S}_2)^2}$

$R_{12}(0)$ — $\overline{(S_1 - \bar{S}_1)(S_2 - \bar{S}_2)}$

$\rho_{12}$ — $\dfrac{R_{12}(0)}{\sqrt{R_1(0) R_2(0)}}$

## 5.3    OPTICAL DATA POINTS

ARGON SCINTILLATION DATA

| Start Time | N | $\rho$ | $R_1(0)$ | $R_2(0)$ | $R_{12}(0)$ | Rate | $\rho_{12}$ |
|---|---|---|---|---|---|---|---|
| 113700 | 32 | 6 | .270 | .520 | .275 | 1 MS | .739 |
| 113825 | 16 | 6 | .575 | .375 | .365 | 1 MS | .786 |
| 114005 | 32 | 8 | .645 | .440 | .375 | 1 MS | .704 |
| 114150 | 32 | 12 | .610 | .420 | .261 | 1 MS | .516 |
| 114420 | 16 | 16 | .375 | .288 | .160 | 1 MS | .487 |
| 114443 | 16 | 16 | .350 | .262 | .125 | 1 MS | .413 |
| 114624 | 32 | 26 | .303 | .228 | .059 | 1 MS | .221 |
| 114905 | 32 | 35 | .450 | .326 | .070 | 1 MS | .183 |
| 115122 | 32 | 50 | .480 | .337 | .039 | 1 MS | .097 |
| 115513 | 32 | 50 | .420 | .290 | .029 | 1 MS | .085 |
| 115910 | 32 | 50 | .480 | .332 | .051 | 1 MS | .128 |
| 120407 | 32 | 35 | .098 | .073 | .026 | 1 MS | .307 |
| 120930 | 16 | 26 | .100 | .081 | .033 | 1 MS | .367 |
| 135336 | 64 | 16 | .012 | .088 | .013 | 1 MS | .400 |
| 131315 | 16 | 14 | .180 | .158 | .061 | 1 MS | .362 |
| 131255 | 32 | 12 | .153 | .153 | .073 | 1 MS | .477 |
| 131450 | 64 | 8 | .160 | .150 | .103 | 1 MS | .665 |
| 131710 | 32 | 6 | .171 | .156 | .126 | 1 MS | .772 |
| 132010 | 32 | 4 | .153 | .146 | .120 | 1 MS | .803 |
| 132100 | 64 | 3 | .118 | .149 | .120 | 1 MS | .811 |
| 132345 | 32 | 2 | .124 | .124 | .116 | 1 MS | .921 |
| 132700 | 32 | 2 | .119 | .124 | .114 | 1 MS | .919 |
| 132916 | 64 | 2 | .116 | .120 | .112 | 1 MS | .949 |
| 133212 | 32 | 3 | .211 | .163 | .168 | 1 MS | .906 |
| 133440 | 32 | 4 | .298 | .185 | .173 | 1 MS | .741 |
| 133540 | 32 | 5 | .318 | .185 | .168 | 1 MS | .693 |
| 133800 | 32 | 5 | .320 | .191 | .200 | 1 MS | .809 |
| 133900 | 32 | 6 | .295 | .165 | .155 | 1 MS | .703 |
| 134013 | 32 | 7 | .360 | .189 | .183 | 1 MS | .702 |
| 134115 | 32 | 8 | .365 | .202 | .180 | 1 MS | .663 |
| 134210 | 64 | 10 | .470 | .240 | .154 | 1 MS | .459 |
| 134330 | 32 | 12 | .430 | .242 | .126 | 1 MS | .397 |
| 134455 | 64 | 14 | .260 | .200 | .088 | 1 MS | .372 |
| 134615 | 32 | 16 | .202 | .171 | .058 | 1 MS | .317 |
| 134730 | 32 | 22 | .218 | .171 | .041 | 1 MS | .212 |
| 135320 | 32 | 50 | .263 | .197 | .018 | 1 MS | .079 |
| 135500 | 64 | 60 | .306 | .213 | .016 | 1 MS | .062 |
| 135720 | 32 | 70 | .310 | .210 | .020 | 1 MS | .078 |
| 135900 | 32 | 80 | .260 | .187 | .014 | 1 MS | .061 |

## 5.5 OPTICAL DATA POINTS (Cont)

### CARBON-DIOXIDE SCINTILLATION DATA

| Start Time | N | $\rho$ | $R_1(0)$ | $R_2(0)$ | $R_{12}(0)$ | Rate | $\rho_{12}$ |
|---|---|---|---|---|---|---|---|
| 113825 | 1 | 8 | .0202 | .0355 | .0184 | 3.33 MS | .691 |
| 111005 | 8 | 10 | .0253 | .0175 | .0191 | 3.33 MS | .860 |
| 114150 | 8 | 14 | .0311 | .0230 | .0191 | 3.33 MS | .725 |
| 114420 | 16 | 18 | .0279 | .0191 | .0089 | 1 MS | .386 |
| 114443 | 16 | 18 | .0202 | .0100 | .0110 | 1 MS | .774 |
| 114631 | 32 | 28 | .0290 | .0161 | .0064 | 1 MS | .389 |
| 114905 | 32 | 37 | 0220 | .0194 | .0133 | 1 MS | .644 |
| 115132 | 32 | 52 | .0268 | .0240 | .0160 | 1 MS | .631 |
| 115330 | 32 | 52 | .0240 | 0195 | .9110 | 1 MS | .506 |
| 115850 | 32 | 52 | .0180 | .0198 | .0079 | 1 MS | .416 |
| 120810 | 32 | 37 | .0140 | .0080 | .0002 | 1 MS | .013 |
| 120930 | 32 | 28 | .0109 | .0110 | .0070 | 1 MS | .639 |
| 130830 | 32 | 14 | .0710 | .090 | .075 | 1 MS | .934 |
| 131015 | 32 | 11 | .0205 | .0150 | .0152 | 1 MS | .841 |
| 131310 | 32 | 14 | .067 | .0460 | .050 | 1 MS | .901 |
| 131450 | 64 | 10 | .070 | .054 | .053 | 1 MS | .895 |
| 131710 | 32 | 8 | .122 | .135 | .121 | 1 MS | .943 |
| 132010 | 32 | 6 | .086 | .090 | .087 | 1 MS | .949 |
| 132310 | 32 | 5 | .075 | .0480 | .059 | 1 MS | .961 |
| 132555 | 32 | 4 | .064 | .047 | .054 | 1 MS | .965 |
| 132710 | 31 | 4 | .076 | .053 | .063 | 1 MS | .993 |
| 132930 | 64 | 4 | .113 | .070 | .082 | 1 MS | .922 |
| 133240 | 16 | 5 | .073 | .092 | .075 | 1 MS | .915 |
| 133445 | 32 | 6 | .064 | .0485 | .046 | 1 MS | .826 |
| 133545 | 32 | 7 | .052 | .078 | .057 | 1 MS | .895 |
| 133800 | 32 | 7 | .082 | .090 | .077 | 1 MS | .896 |
| 133900 | 32 | 8 | .061 | .082 | .061 | 1 MS | .863 |
| 134025 | 32 | 9 | .037 | .064 | .040 | 1 MS | .822 |
| 134115 | 32 | 10 | .071 | .128 | .080 | 1 MS | .839 |
| 134210 | 64 | 12 | .063 | .081 | .051 | 1 MS | .714 |
| 134330 | 32 | 14 | .075 | .110 | .069 | 1 MS | .760 |
| 134455 | 64 | 16 | .078 | .093 | .070 | 1 MS | .813 |
| 134615 | 32 | 18 | .067 | .075 | .067 | 1 MS | .829 |
| 134730 | 32 | 24 | .069 | .080 | .058 | 1 MS | .781 |
| 134845 | 32 | 28 | .074 | .089 | .064 | 1 MS | .789 |
| 135000 | 32 | 32 | .063 | .061 | .047 | 1 MS | .758 |
| 135320 | 32 | .046 | .048 | .041 | .0168 | 1 MS | .379 |
| 135500 | 64 | .05 | .035 | .051 | .021 | 1 MS | .397 |
| 135720 | 32 |  | .071 | .066 | .044 | 1 MS | .563 |
| 135900 | 32 |  | .155 | .102 | .073 | 1 MS | .581 |

# 6. DATA ANALYSIS

## 6.1 VERTICAL PROFILE OF TURBULENCE

Most of the estimates of the vertical profile of turbulence have been based upon measurements of stellar scintillation and astronomical seeing. Hufnagel and Stanley[1] proposed one of the first and best-known profiles and in 1966 Hufnagel[2] revised the profile (Figure 6-1). Fried[3] proposed a mathematical model of the profile of atmospheric turbulence and has used it extensively in his numerous papers on optical propagation (Figure 6-2).

Due to the presence of a sharp temperature gradient at the tropopause, most authors have assumed that the tropopause is the major source of stellar scintillation effects. Analysis by Tatarski[4] based upon measurements of aperture averaging by Protheroe[5] and stellar covariance measurements by Keller[6] tend to substantiate this hypothesis. Titterton[7] and Brookner[] have pointed out that Fried's model of the atmosphere overestimates the strength of turbulence in the first few kilometers of the atmosphere while not properly accounting for turbulence near the tropopause. The major points of evidence indicating a narrow band of high turbulence at or near the tropopause are the size of the correlation length of stellar scintillation and the correlation of the power spectral density of scintillation to wind speed at the tropopause. However, wide variations in the size of the stellar scintillation correlation distance (from 3 to 10 cm) make the determination of the altitude of the scintillation generating layers of turbulence difficult.

The strength of turbulence is normally specified by the refractive index-structure constant

$$C_N^2 = \frac{D_N(\rho)}{\rho^{2/3}} = \frac{\langle (N_1 - N_2)^2 \rangle}{\rho^{2/3}} \qquad l_o < \rho < L_o \qquad (1)$$

where $N_1$ and $N_2$ are the refractive indices at two points separated by a distance $\rho$. Here we have assumed the Kolmogorov[9] similarity theory of turbulence to hold for separation $(\rho)$ between the inner $(l_o)$ and outer scale $(L_o)$. The refractive index of air at optical frequencies may be expressed as a function of the properties of a gas and the wavelength through the relation

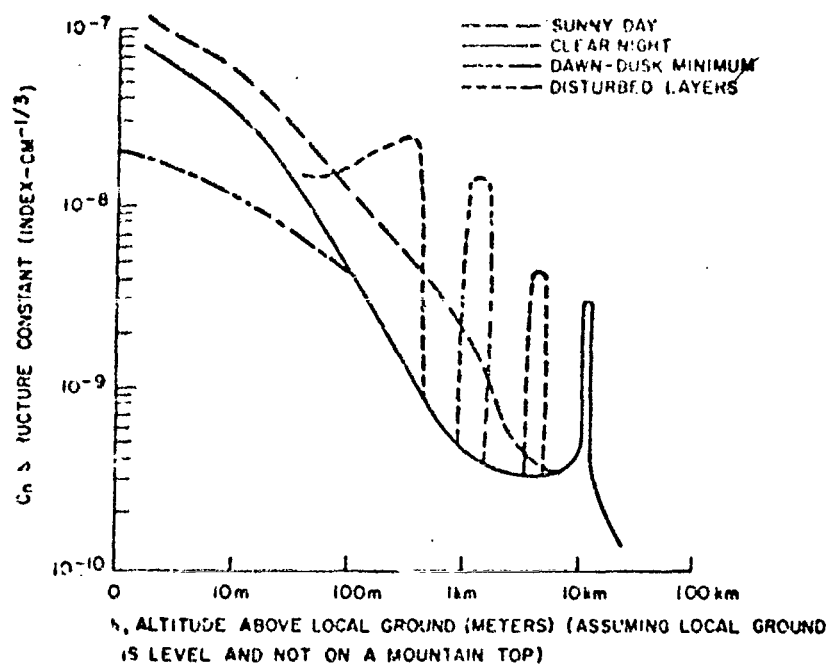$$N = 1.0 + \frac{77.6 \, P}{T} \left[ 1 + \frac{0.0075}{\lambda^2} \right] \times 10^{-6} \qquad (2)$$
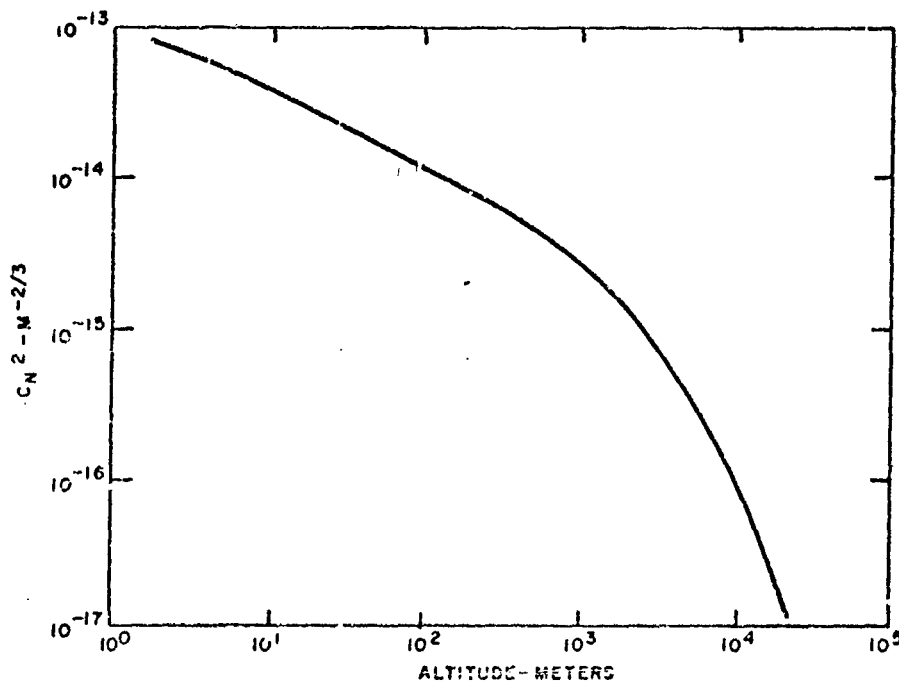
Figure 6-1. Hufnagel and Stanley Profile, Revised 1966

Figure 6-2.   Fried's Model

where P is pressure in millibars, T is absolute temperature in $^\circ K$, and $\lambda$ is wavelength in $\mu m$. Pressure fluctuations are rapidly damped out[10] and, therefore, refractive index changes are mainly due to microthermal fluctuations.

Defining the temperature structure constant as

$$C_T^2 = \frac{D_T(\rho)}{\rho^{2/3}} = \frac{\langle (T_1 - T_2)^2 \rangle}{\rho^{2/3}} \qquad l_o < \rho < L_o \qquad (3)$$

we see that it can be easily converted to $C_N^2$;

$$C_N^2 = \left[ \frac{77.6P}{T^2} \left( 1 + \frac{0.0075}{\lambda^2} \right) \right]^2 C_T^2 \times 10^{-12} \qquad (4a)$$

$$\approx \left[ \frac{80P}{T^2} \right]^2 C_T^2 \times 10^{-12} \qquad (4b)$$

6-3

Equations 3 and 4a provide the most direct method by which to obtain the vertical profile of turbulence. The temperature structure constant can be measured with a pair of low-inertia-resistance thermometers which are part of a Wheatstone bridge. The output voltage of the bridge, which is proportional to the temperature difference between the thermometers, may be amplified and the mean square obtained with suitable electronic circuits. $D_T(\rho)$ combined with data on mean pressure and temperature may then be used to compute $C_N^2$. Krechmer[11], Tatarski[12], and Lawrence[13] have used this method to obtain data in the first few kilometers of the atmosphere employing both aircraft and tethered balloons to elevate their instruments. Unfortunately the data extends only up to 3.5 km.

In order to compare the results of the optical measurements with theory. an instrument to measure the vertical profile of turbulence was constructed and flown as part of the balloon payload*. The device consists of a pair of low-inertia-resistance thermometers in a Wheatsone bridge with suitable processing electronics to derive $C_T^2$. The device was attached to a standard MD-317A/AMT-12A radiosonde which was suspended on a 300-m cable below the main payload to avoid the turbulence effects caused by the rising balloon. The output of the resistor thermometer system was connected to one of the data channels of the radiosonde and data w s telemetered to the ground in a standard telemetry format. Mean pressure and temperature were obtained from the standard radiosonde sensors**. The profile of $C_N^2$ obtained is shown in Figure 6-3.

Because of the rapid ascent of the balloon (1000 ft/min or 5 m/sec) the radiosonde provided only a few data points within the first few hundred meters. Therefore, Tatarski's[14] method of mean temperatures was used to supplement the thermosonde data. From Figure 16 of Reference 10 we have approximated Tatarski s data by the following equations:

Stable Stratification (Temperature Inversion – Increase of mean temperature with height)

$$C_T^2 = 4 \times 10^{-2} \left| K^{2/3} h^{-1/3} T_* \right| \tag{5}$$

Unstable Stratification (Decrease of mean temperature with height)

$$C_T^2 = 5.76 K^{4/3} h^{-2/3} T_*^2 \tag{6}$$

*See Section 3.2        **See Appendix A

Where 
$$T_* = \frac{\bar{T}_1 - \bar{T}_2}{\log_{10}\left[\frac{h_1}{h_2}\right]}$$

$$K = 0.4 \tag{7}$$

Therefore

Stable Stratifraction

$$C_T^2 = 2.17 \times 10^{-2} \left| h^{-1/3} T_* \right| \tag{8}$$

$$C_N^2 = 1.47 \times 10^{-14} \left| h^{-1/3} T_* \right| \quad (P = 876 \text{ mb}, \ T = 280^\circ K) \tag{9}$$

Unstable Stratifraction

$$C_T^2 = 1.73 z^{-2/3} T_*^2 \tag{10}$$

$$C_N^2 = 1.17 \times 10^{-12} \ z^{-2/3} T_*^2 \quad (P = 876 \text{mb}, \ T = 280^\circ K) \tag{11}$$

The experiment was performed under conditions of stable stratifraction with an inversion at about 180 m, as shown by the vertical dashed line of Figure 6-3. Therefore, equation 5 was used to calculate $C_N^2$ up to an altitude of about 80 m. The results of this analysis are shown in Table 6-1 and as crosses in Figure 6-3.

Table 6-1

| h (m) | log h | $h^{-1/3}$ | $T_* (^\circ K)$ | $C_T^2 (Km^{-2/3})$ | $C_N^2$ |
|-------|-------|------------|------------------|---------------------|---------|
| 10 | 1.00 | 0.464 | 1.40 | $1.47 \times 10^{-2}$ | $1.00 \times 10^{-14}$ |
| 20 | 1.30 | 0.368 | 2.92 | $2.34 \times 10^{-2}$ | $1.59 \times 10^{-14}$ |
| 40 | 1.60 | 0.293 | 5.92 | $3.77 \times 10^{-2}$ | $2.56 \times 10^{-14}$ |
| 60 | 1.78 | 0.255 | 10.3 | $5.70 \times 10^{-2}$ | $3.89 \times 10^{-14}$ |
| 80 | 1.90 | 0.231 | 12.7 | $6.35 \times 10^{-2}$ | $4.31 \times 10^{-14}$ |

The profile of turbulence has been approximated by a mathematical model shown by the dashed line, which obeys the equation

$$C_N^2 = C_{N_0}^2 \ h^{-1/3} e^{-h/h_o} \tag{12}$$

REFRACTIVE-INDEX-STRUCTURE CONSTANT AS A
FUNCTION OF HEIGHT ABOVE GROUND LEVEL.
1:00 AM MST 21 OCTOBER 1970 AT HOLLOMAN AFB
NEW MEXICO. BAPE I EXPERIMENT

+ DATA POINTS OBTAINED BY TATARSKI'S
  METHOD OF MEAN TEMPERATURES

● DATA POINTS OBTAINED FROM THERMOSONDES
  MEASUREMENT

— — EXPERIMENTAL DEPENDENCE OF $C_n^2$ UPON
  ALTITUDE

– – – MATHEMATICAL MODEL USED TO REPRESENT
  EXPERIMENTAL DATA

$$C_n^2 = C_{no}^2 \, h^{-1/3} \, e^{-h/h_o}$$

$$C_{no}^2 = 4.0 \times 10^{-14} \quad m.^{-2/3} \qquad h_o = 1000 \, m.$$

GROUND LEVEL $1.2 \times 10^3$ m. ABOVE MSL
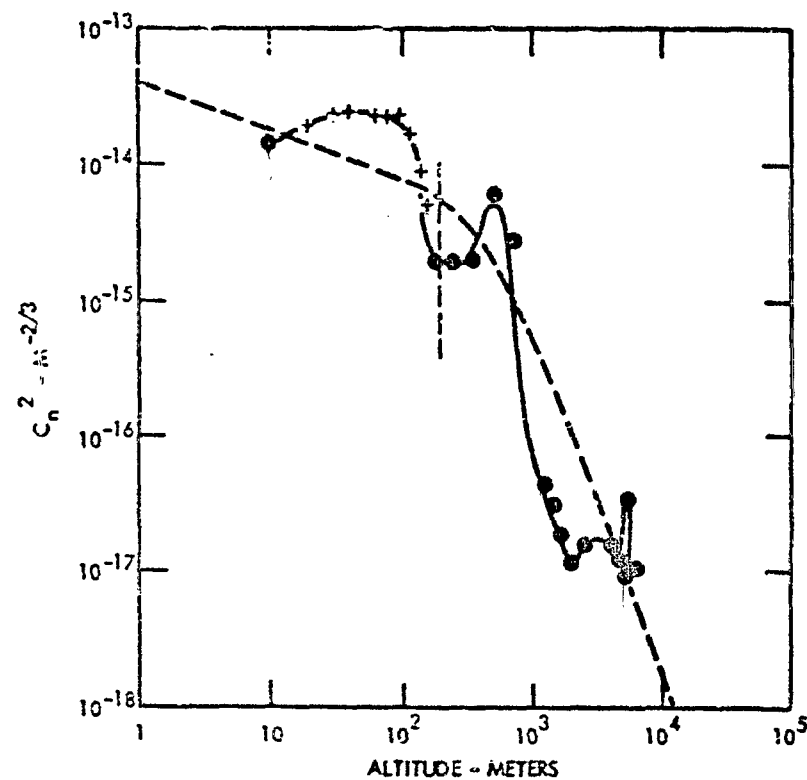


Figure 6-3

where $C_{No}^2 = 4 \times 10^{-14} \ m^{-2/3}$

$h_o = 1000 \ m.$

The value of $C_{No}^2$ represents the ordinate of the model at $h = h_o$, although it has no physical significance since the refractive-index structure constant is not reliable so close to the ground. The scale height is the break point in the model, which for stable stratifraction normally occurs at the temperature inversion.

Two integrals of $C_N^2$ which will be of importance in later calculations of the scintillation statistics are

$$\int_o^\infty C_N^2(h) \ dh = C_{No}^2 h_o^{2/3} \ \Gamma(^2/_3) = 1.30 \ C_{No}^2 \ h_o^{2/3} \ (m^{1/3}) \tag{13}$$

$$= 5.20 \times 10^{-12} \ m^{1/3}$$

and

$$\int_o^\infty C_N^2(h) \ h^{5/6} \ dh = C_{No}^2 h_o^{3/2} \ \Gamma(^3/_2) = 0.89 \ C_{No}^2 h_o^{3/2} \ (m^{7/6}) \tag{14}$$

$$= 1.13 \times 10^{-9} \ m^{7/6}$$

Wind speed and mean temperature as a function of altitude are shown in Appendix A. No thermosonde data is available above 6 kilometers; therefore, Hufnagel's model for the tropopause turbulence is used with the integral over the tropopause equal to

$$\int_{trop} C_N^2(h) \ dh = 4.31 \times 10^{-13} \quad m^{1/3}$$

The final model of the atmosphere used for calculation of optical propagation statistics becomes

$$C_N^2 = C_{No}^2 \ h^{-1/3} \ e^{-h/h_o} + \delta(h-h_p) \ C_{Np}^+ \tag{15}$$

where

$C_{No}^2 = 4.0 \times 10^{-14} \ m^{-2/3}$     $h_o = 1000 \ n.$

$C_{Np}^+ = 4.3 \times 10^{-13} \ m^{1/3}$     $h_p = 13.2 \times 10^3 \ m$

# REFERENCES

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## 6.2 LOG-AMPLITUDE VARIANCE AND SCINTILLATION

The strength of scintillation is often specified by the normalized power variance at the receiver;

$$\sigma_p^2 = \frac{<(P-\overline{P})^2>}{\overline{P}^2} \tag{1}$$

While this is a simple parameter to measure, it is not a convenient term to handle statistically. Because the power is log-normally distributed (that is the logarithm of power is normally distributed) as a consequence of the central limit theorem, it is more convenient to make calculations based on

$$\ln \frac{P}{P_0}$$

which has a normal distribution. This parameter, sometimes referred to as log-power or log intensity, may be analyzed using the normal distribution tables, and its variance is simply related to the normalized power variance

$$<(\ln P/P_0)^2> = \ln (1 + \sigma_p^2) \tag{2}$$

Since most of the theoretical analysis of optical propagation is done in terms of phase and amplitude, the analyst uses the term log-amplitude

$$\ln \frac{A}{A_0}$$

which also has a normal distribution. The variance of this parameter is easily related to the log-power (or log-intensity) variance

$$<(\ln A/A_0)^2> = \frac{1}{4} <(\ln P/P_0)^2> = \frac{1}{4} \ln (1 + \sigma_p^2) \tag{3}$$

In the case where $\sigma_p^2 << 1$, then

$$\ln (1 + \sigma_p^2) \approx \sigma_p^2 \tag{4}$$

and we can make the approximation

$$<(\ln A/A_0)^2> \approx \frac{\sigma_p^2}{4} \tag{5}$$

Log-amplitude variance has been represented by numerous different symbols by various authors, but we will use the terminology of Fried[1] and use $C_1(0)$ or $\sigma_1{}^2$. Small letter l will be used to represent log-amplitude.

If a laser beam is transmitted from an optical system which allows for the adjustment of the aperture and radius of curvature of the emergent wavefront, the scalar wave function at the optical system output ($z = 0$) is

$$\mu_0 (r)\big|t = 0 = \exp\left[-\frac{\rho^2}{2\sigma_0{}^2} + i k \rho^2/2R\right] \tag{6}$$

where $k = 2\pi/\lambda$, $\rho$ is the radial component of r, and z is the axial component of r. Let the beam have a central amplitude of unity and a normal radial distribution with variance $\alpha_0{}^2$; then from Schmeltzer[2]

$$C_1(0) = -(k^2/8\pi)\ \mathrm{Re}\left(\int_0^z ds \int_0^\infty d\sigma\phi\ (\sigma^{1/2}; s) \right.$$
$$\times \exp\left[\sigma/4\ \gamma(z, s)\right]\ \left\{\exp\left[(\sigma/_4)\ \gamma(z, s)\right]\right.$$
$$\left.\left. - \exp\left[(\sigma/_4)\gamma'(z, s)\right]\right\}\right) \tag{7}$$

R has been chosen such that a positive R means a divergent laser beam, and the integration is along the propagation path from $s = 0$ at the source to $s = z$ at the receiver. From the Kolmogorov theory of turbulence[3]

$$\phi(\sigma; s) = 8.16\ C_N{}^2\ \sigma^{-11/3} \tag{8}$$

The gamma function is defined

$$\gamma(z, s) = \left[2(z - s)/ik\right]\ \left[(s - ik\alpha^2)/(z - ik\alpha^2)\right] \tag{9}$$

where
$$\frac{1}{\alpha^2} = \frac{1}{\sigma_0{}^2} - \frac{ik}{R} \tag{10}$$

Combining equation 7 and equation 8 to obtain an expression for log-amplitude variance

$$C_1(0) = \frac{8.16}{8\pi} k^2\ \mathrm{Re}\left\{\int_0^z ds C_N{}^2 (s)\int_0^\infty d\sigma\ \sigma^{-11/6}\ \left(\exp\left\{(\sigma/2)\ \mathrm{Re}\left[\gamma(z, s)\right]\right\} - \exp\right.\right.$$
$$\left.\left.\left[(\sigma/2)\ \gamma(z, s)\right]\right)\right\} \tag{12}$$

For $\alpha_0 = 0$  $\gamma(z, s) = \frac{2(z-s)}{ik} \left(\frac{s}{z}\right)$ (13)

$$C_1^s(0) = \frac{8.16}{8\pi} \Gamma(-5/6) k^2 \text{Re} \left\{ \int_0^z ds\, C_N^2(s) \left\{ \left[-1/2\, \text{Re}\gamma(z,s)\right]^{5/6} - \left[-1/2\gamma(z,s)\right]^{5/6} \right\} \right\}$$

$$= \frac{8.16}{4\pi} \Gamma(-5/6) k^{7/6} \text{Re} \left\{ \int_0^z ds\, C_N^2(s) \left[(z-s)\left(\frac{s}{z}\right)\right]^{5/6} (i)^{5/6} \right\}$$ (14)

For   $i^{5/6} = 0.259 + 0.965\, i$   $\Gamma(-5/6) = 6.68$

$$C_1^s(0) = 0.56 k^{7/6} \int_0^z ds\, C_N^2(s) \left[(z-s)\left(\frac{s}{z}\right)\right]^{5/6}$$ (15)

The maximum value of $\left[(z-s)\left(\frac{s}{z}\right)\right]$ occurs when $s = z/2$ indicating turbulence halfway between source and receiver is most effective in causing scintillation. When $z \gg s_m$ where $s_m$ is the distance at which $C_N^2(s) \to 0$.

$$C_1^s(0) = 0.56 k^{7/6} \int_c^{s_m} C_N^2(s)\, s^{5/6}\, ds$$ (16)

If the propagation is in a vertical direction to a receiver at altitude H and zenith angle $\Theta$, the log-amplitude variance becomes

$$C_1^s(0) = 0.56 k^{7/6} \sec^{11/6}\Theta \int_0^H C_N^2(h) \left[(H-h)\frac{h}{H}\right]^{5/6} dh$$ (17)

Or for $H \gg h_m$ where $h_m$ is the altitude at which

$$C_N^2(h) \to O$$

$$C_1^s(0) = 0.56 k^{7/6} \sec^{11/6}\Theta \int_0^{h_m} C_N^2(h)\, h^{5/6}\, dh$$ (18)

It should be noted that there is a reciprocity between plane wave space to earth propagation and spherical wave earth to space propagation. Because of this reciprocity, the scintillation of a star is the same as that of a ground-based laser source viewed from space, and equation 18 is the same as Tatarski's[4] equation 8.17. This fact was used in the GEOS-II laser detector experiment in order to relate upward propagation to downward propagation. This reciprocity is also an important factor in the analysis of this experiment.

For nonpoint sources ($\alpha_o > 0$) the expression for log-amplitude variance is more complex. This problem has been analyzed by Fried[6] who finds that the spherical-wave, log-amplitude variance may be modified by a correction factor (Figure 6-4) which is a function of the normalized source size $\Omega$.

$$\Omega = k \alpha_o^2 / h_o \sec \Theta \tag{19}$$

The sources used in the experiment could not be made true point sources. Therefore, the normalized source size was made equal for both transmitters. This required that the standard deviations of beam profiles have the ratio

$$\frac{\alpha_{oa}}{\alpha_{ob}} = \left[ \frac{k_a}{k_b} \right]^2 = \left[ \frac{\lambda b}{\lambda a} \right]^2 = \left[ \frac{10.6 \times 10^{-6}}{0.5145 \times 10^{-6}} \right]^2 = 4.55 \tag{20}$$

where subscript a refers to the argon laser ($\lambda = 0.5145 \times 10^{-6}$ m) and b to the carbon-dioxide laser ($\lambda = 10.6 \times 10^{-6}$ m). The laser outputs were made 6 mm (argon) and 30 mm (carbon-dioxide) by passing them through optical systems which were adjusted to also provide a divergence of approximately one milliradian. The optical transmitter system is described in section 3.4.1.

The normalized source size for both transmitters for the turbulence profile of the experiment at a zenith angle of $45^\circ$ was

$$\Omega = k \alpha_o^2 / h_o \sec \Theta = \frac{5.92 \times 10^5 (0.03)^2}{1000 \times 1.4} = 0.381 \tag{21}$$

which resulted in an aperture averaging correction factor of 0.5. The balloon was flown at an altitude of 28.0 km above ground level, and was at a zenith angle of approximately $45^\circ$ during the period data was taken. The transmitter was located on a level plain 1.2 km above mean sea level.
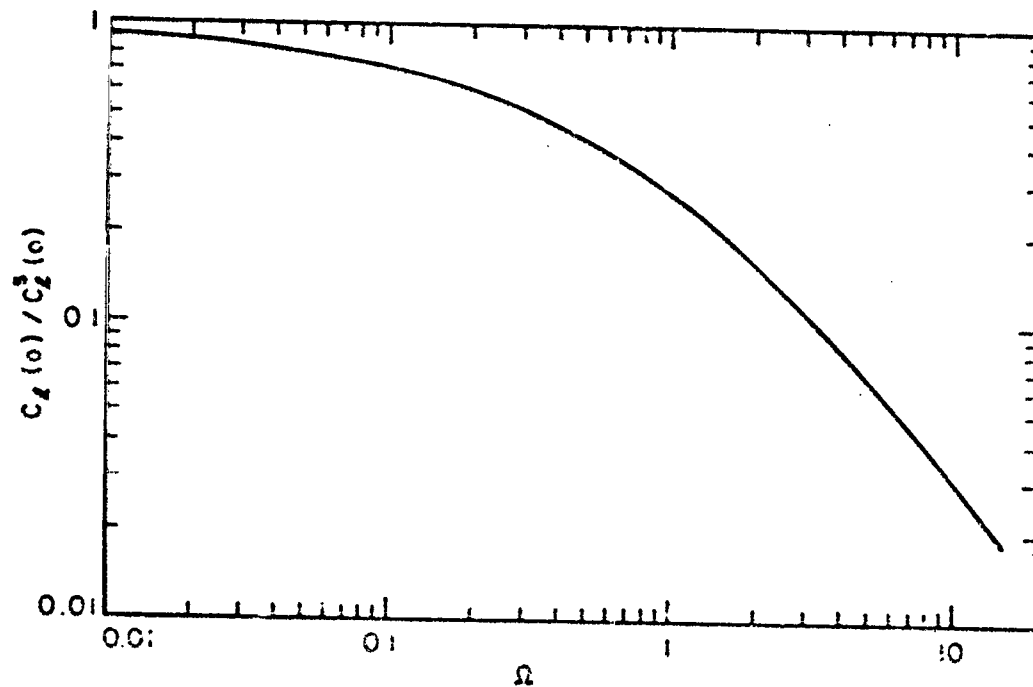
Figure 6-4. Fried's Aperture Correction Factor

1062 6.2-1

Using these factors and the turbulence profile of the previous section we get (altitude of the receiver was 28.0 km above ground level) the spherical wave log-amplitude variances

$$C_l(0) = 0.56 \, k^{7/6} \sec^{11/6} \Theta \int_0^\infty C_N^2(h) \left[(H-h)\frac{h}{H}\right]^{5/6} dh$$

$$\approx 0.56 \, k^{7/6} \sec^{11/6} \Theta \int_0^\infty C_{No}^2 \, h^{-1/3} e^{-h/h_o} \, h^{5/6} \, dh$$

$$\approx 1.10 \, k^{7/6} \left[0.89 C_{No}^2 h_o^{3/2}\right] = 1.10 \, k^{7/6} \left[1.13 \times 10^{-9}\right]$$

$$= 1.24 \times 10^{-9} \, k^{7/6}$$

$$= 1.24 \times 10^{-9} \times 1.81 \times 10^8 = 0.224 \text{ (argon)} \tag{22a}$$

$$= 1.24 \times 10^{-9} \times 5.44 \times 10^6 = 0.00674 \text{ (carbon-dioxide)} \tag{22b}$$

After correction for aperture averaging

$$C_l(0) = 0.59 \, (0.224) = 0.112 \text{ (argon)} \tag{23a}$$

$$C_l(0) = 0.50 \, (0.00674) = 0.00337 \text{ (carbon-dioxide)} \tag{23b}$$

To these variances we must add the scintillation caused by the tropopause

$$C_l(0)_{trop} = 0.56 \, k^{7/3} \sec^{11/6} \Theta \, C_{np}^+ \left[(H-h_p)\frac{h_p}{H}\right]^{5/6}$$

$$= 1.10 \, k^{7/6} \times 4.3 \times 10^{-13} \left[(28.0 \times 10^3 - 13.2 \times 10^3)\frac{13.2 \times 10^3}{28.0 \times 10^3}\right]^{5/6}$$

$$= 6.62 \times 10^{-10} \, k^{7/6}$$

$$= 6.62 \times 10^{-10} \times 1.81 \times 10^8 = 0.120 \text{ (argon)} \tag{24a}$$

$$= 6.62 \times 10^{-10} \times 5.44 \times 10^6 = 0.00360 \text{ (carbon-dioxide)} \tag{24b}$$

The total log-amplitude variance due to the turbulence near the ground plus the turbulence at the tropopause becomes

$$C_l(0) = 0.112 + 0.120 = 0.232 \text{ (argon)} \tag{25a}$$

$$= 0.00337 + 0.0036 = 0.00697 \text{ (carbon-dioxide)} \tag{25b}$$

A pair of sensors aboard the balloon payload were used to monitor the irradiance caused by the lasers. Instantaneous irradiance levels were telemetered to the ground and recorded on magnetic tape. Later, the tape was played back in 30-second intervals into a Hewlett-Packard correlator from which sample values of $\sigma_p{}^2$ were obtained. Normalized power variances were converted to log-amplitude variances by equation 3. Hufnagel[7] has predicted that the strength of scintillation as measured by log-amplitude variance should itself be a log-normal parameter. Therefore, $\ln C_1(0)$ was plotted versus cumulative probability on normal graph paper to check the distribution of $C_1(0)$. A normal distribution plotted in this manner will appear as a straight line. Results are shown in Figures 6-5 and 6-6. The parameters of the distributions are listed below:

|  | 0.5145 $\mu m$ | 10.6/$\mu m$ |
|---|---|---|
| $\overline{\ln C_1(0)}$ | -1.8 | -5.4 |
| $\sigma_{\ln C_1}(0)$ | 0.6 | 0.6 |

The magnitude of $C_1(0)$ should, from equation 15, be proportional to the 7/6 power of the wavenumber. Therefore

$$\frac{[C_1(0)]_{0.5145\ \mu m}}{[C_1(0)]_{10.6\ \mu m}} = \left[\frac{10.6 \times 10^{-6}}{0.5145 \times 10^{-6}}\right] = 34.1 \tag{26}$$

From the measured log-amplitude variances

$$\frac{\text{Median}[(C_1(0)]_{0.5145\ \mu m}}{\text{Median}[(C_1(0)]_{10.6\ \mu m}} = \frac{e^{-1.8}}{e^{-5.4}} = 36.9 \tag{27}$$

The mean $C_1(0)$ from the data are*

$$\overline{C_1(0)} = e^{-1.8 + \frac{1}{2}(.6)^2} = e^{-16.2} = 0.198 \quad (0.5145\ \mu m) \tag{28a}$$

$$\overline{C_1(0)} = e^{-5.4 + \frac{1}{2}(.6)^2} = e^{-5.22} = 0.0054 \quad (10.6\ \mu m) \tag{28b}$$

---

*For a log-normal variable $\mu = \exp\left[k + \frac{\lambda^2}{2}\right]$ where $\mu$ is the mean of the log-normal variable and k and $\lambda^2$ are the mean variance of the corresponding normal variables.
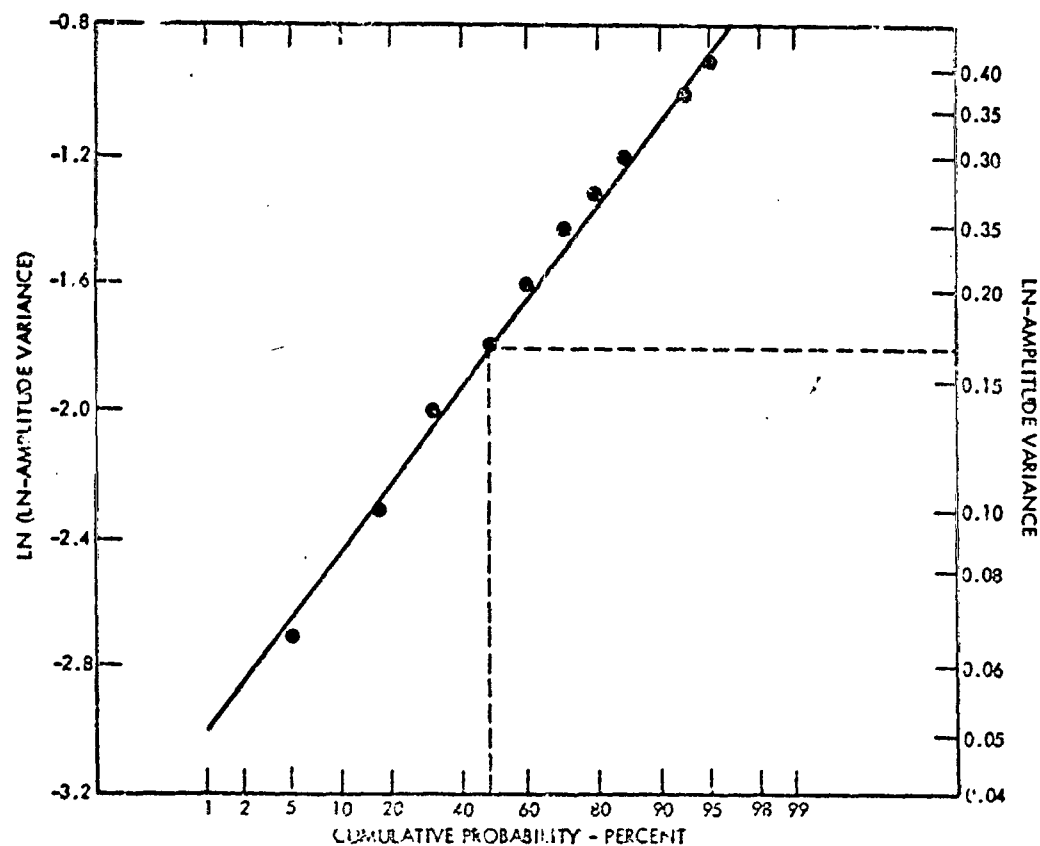
CUMULATIVE PROBABILITY* OF LOG-AMPLITUDE VARIANCE FOR ARGON WAVELENGTH (5145 A) FOR A 2½ HOUR PERIOD. 4:30 – 7:00 AM MST 21 OCTOBER 1970 AT HOLLOMAN AFB, NEW MEXICO. BAPE I EXPERIMENT

*DATA PLOTTED ON NORMAL PROBABILITY SCALE. STRAIGHT LINE INDICATES NORMAL DISTRIBUTION.

Figure 6-5. Cumulative Probability of Log-amplitude Variance for Argon Wavelength
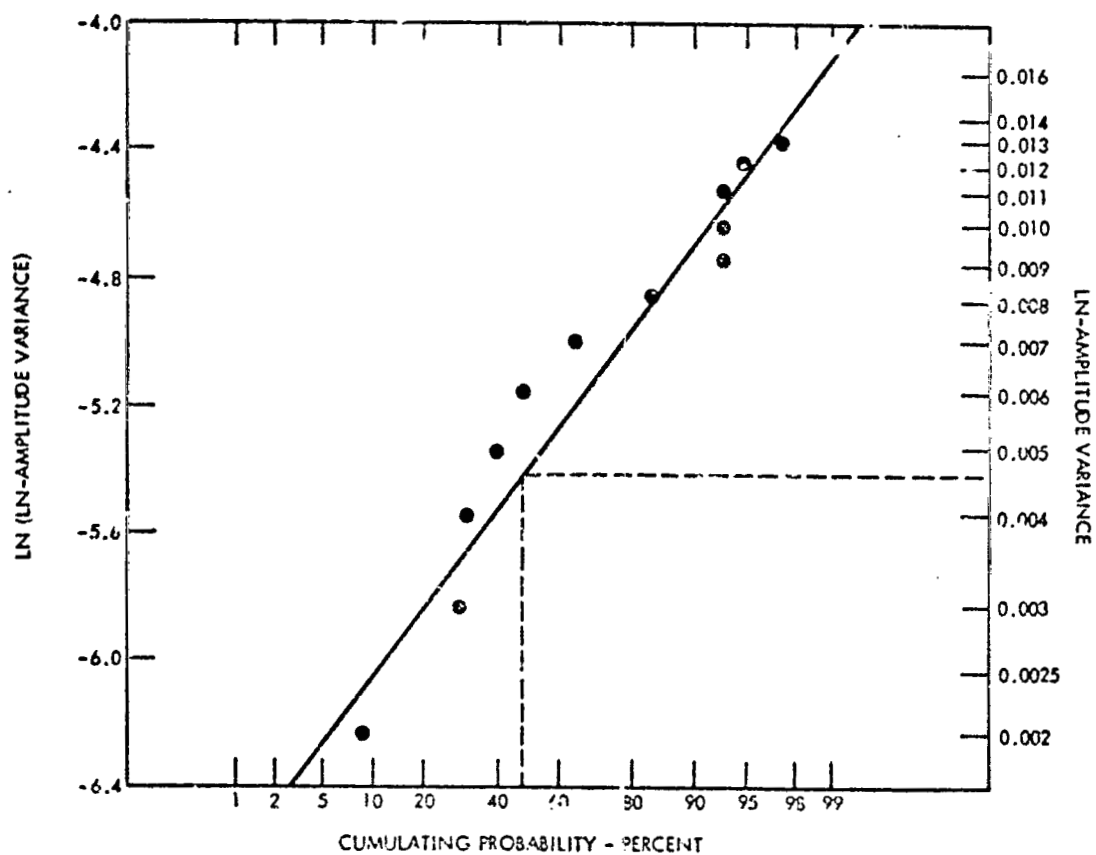
CUMULATIVE PROBABILITY* OF LOG-AMPLITUDE VARIANCE FOR $CO_2$ WAVELENGTH
(10.6 micrometers) FOR A 2½ HOUR PERIOD. 4·30 - 7:00 AM MST 21 OCTOBER 1970 AT
HOLLOMAN AFB, NEW MEXICO. BAPE I EXPERIMENT

*DATA PLOTTED ON NORMAL PROBABILITY SCALE. STRAIGHT LINE INDICATES
 NORMAL DISTRIBUTION.

Figure 6-6. Cumulative Probability of Log-amplitude Variance for $CO_2$ Wavelength

Experimental values of $C_1(0)$ determined by optical measurement are compared to values computed from the turbulence profile in the following table:

|  | Experimental | Computed | Exp/Computed |
|---|---|---|---|
| $C_1(0)$ 0.5145 $\mu m$ | 0.198 | 0.232 | 0.854 |
| $C_1(0)$ 10.6 $\mu m$ | 0.00540 | 0.00697 | 0.775 |

The central-limit theorem predicts that the distribution of irradiance should be log-normal. This was checked in Figures 6-7 and 6-8 by plotting the cumulative probability of the logarithm of a normalized signal on a normal distribution scale. The straight line behavior of both of these graphs again indicates a log-normal distribution.

In order to assure that the experimental values of variance were not causing a scatter of the results due to insufficient sample lengths, an analysis of the statistical error was performed.

Standard statistical techniques provide a means to estimate the effect of finite data sample length. For a number, N, of independent observations of a normally distributed random variable, x, it is known that the sample variance obeys a chi-square distribution with n degrees of freedom.

$$\frac{n S^2}{\sigma_x^2} = X_n^2 \qquad \begin{array}{l} n = N - 1 \\ S^2 = \text{sample variance} \\ \sigma_x^2 = \text{actual variance} \\ X_n^2 = \text{chi-square random variable} \end{array} \qquad (29)$$

of chi-square determine the $1 - \alpha$ confidence interval for sample variance,

$$\frac{n}{X_{n;\,\alpha/2}^2} \; S^2 \leq \sigma_x^2 \leq \frac{n}{X_{n;\,1-\alpha/2}^2} \; S^2 \qquad (30)$$
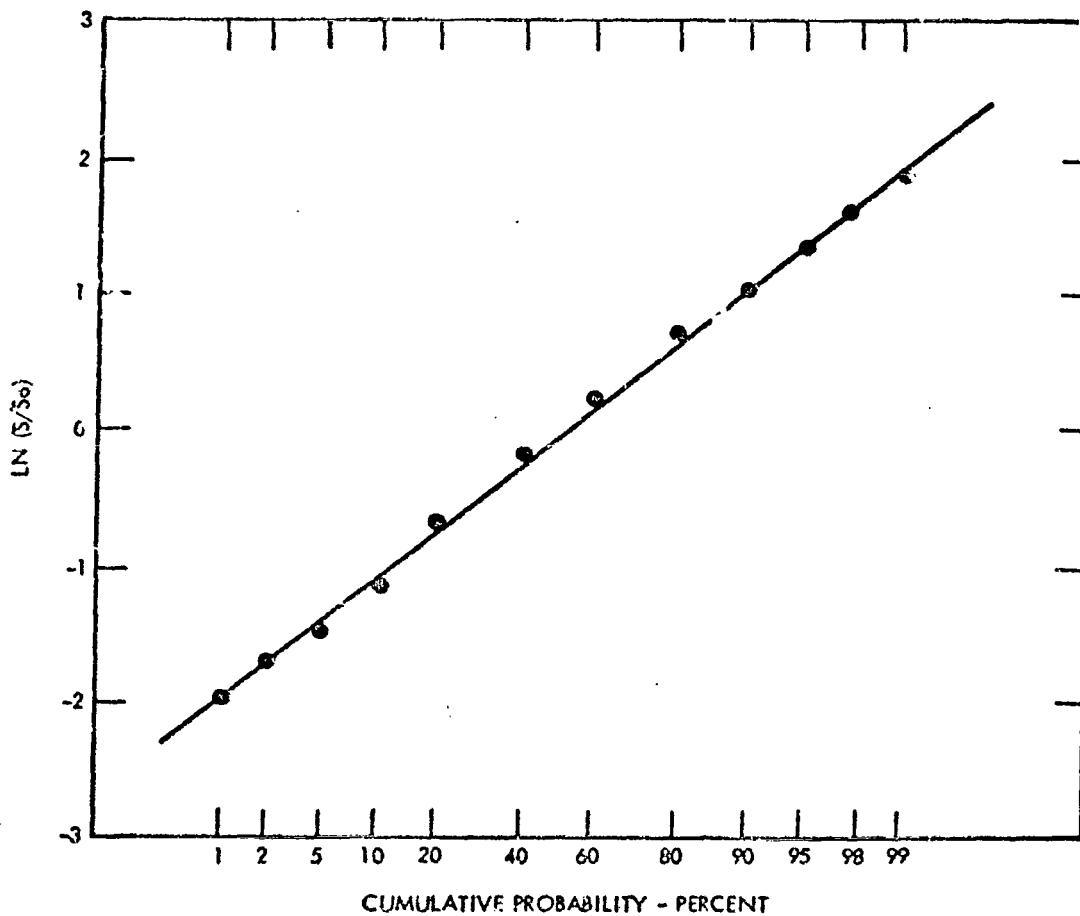
THE LOGARITHM OF NORMALIZED RECEIVED SIGNAL(S) FOR THE ARGON WAVELENGTH
(5145 A) AS A FUNCTION OF CUMULATIVE PROBABILITY ON NORMAL PROBABILITY PAPER.
49:05 MST 21 OCTOBER 1970 AT HOLLOMAN AFB, NEW MEXICO. BAPE I EXPERIMENT

VARIANCE 0.72
LOG-AMPLITUDE VARIANCE ≈0.18
So - MEAN SIGNAL LEVEL

Figure 6-7.  Logarithm of Normalized Received Signals for Argon Wavelength

CUMULATIVE PROBABILITY - PERCENT
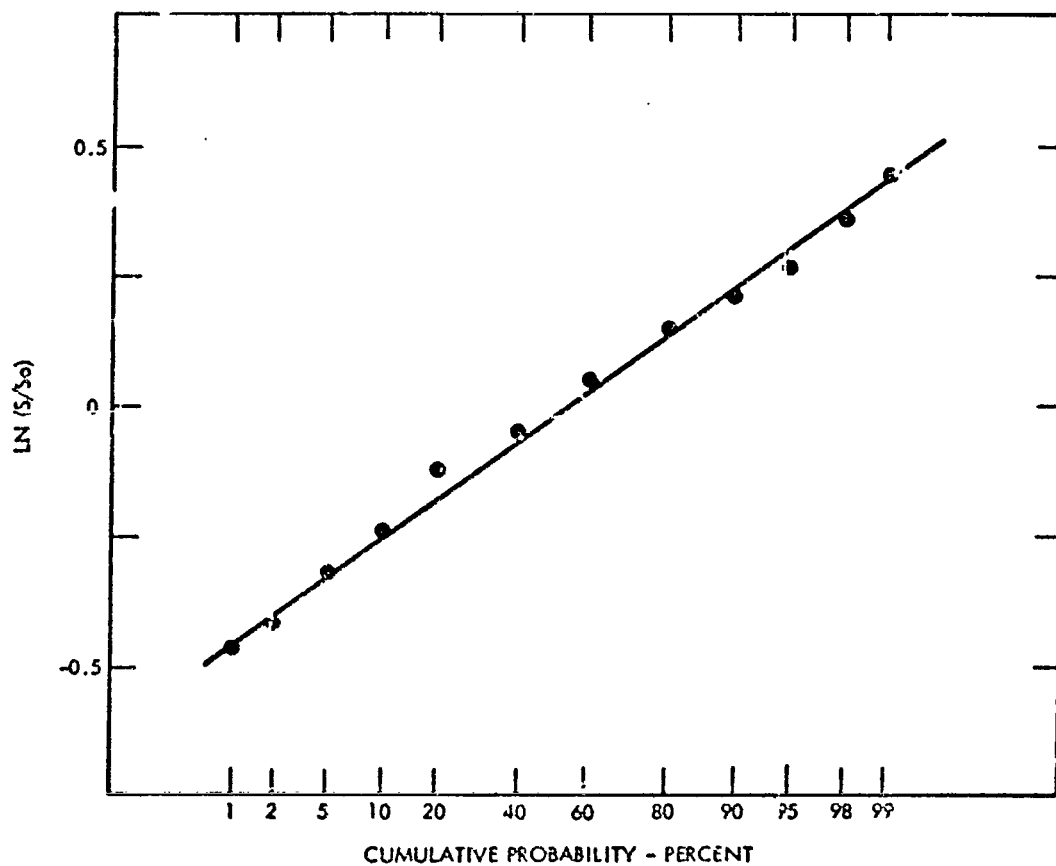
THE LOGARITHM OF NORMALIZED RECEIVED SIGNAL(S) FOR THE $CO_2$ WAVELENGTH
(10.6 micrometers) AS A FUNCTION OF CUMULATIVE PROBABILITY ON NORMAL
PROBABILITY PAPER. 6:47:30 AM MST 21 OCTOBER 1970 AT HOLLOMAN AFB, NEW
MEXICO. BAPE I EXPERIMENT

VARIANCE 0.04
LOG-AMPLITUDE VARIANCE $\approx 0.01$
$S_0$ - MEAN SIGNAL LEVEL

Figure 6-8. Logarithm of Normalized Received Signals for $CO_2$ Wavelength

that is, the actual variance $\sigma_x^2$ is known to fall within the above interval with a confidence of 100 $(1 - \alpha)$ percent. The key parameter that determines the above interval is N, the number of independent samples.

$$N = 2 B_{eq} T \tag{31}$$

where

$B_{eq}$ = equivalent white noise bandwidth

$T$ = record length

$B_{eq}$ in turn can be estimated from

$$B_{eq} = \frac{R_x(0)}{2 \int_{\infty}^{\infty} R_x(\gamma) \, d_\gamma} \tag{32}$$

where

$R_x(\gamma)$ = autocorrelation for random variable

Typical values of $R_x(\gamma)$ for argon and $CO_2$ data lead to estimates for $B_{eq}$ and in turn N for a given sample record length, T. The value of chi-square for a particular N and is obtained from standard tables. Results are given below for a 90-percent confidence interval for each signal and a sample record length of 30 seconds.

### Statistical Error Analysis

| Data | $B_{eq}$ (Hz) | N | $\alpha$ | $\chi_n^2; \alpha/2$ | $\chi_n^2; 1-\alpha/2$ |
|------|------|------|------|------|------|
| Argon | 150 | 10,500 | 0.1 | 10,740 | 10,263 |
| $CO_2$ | 35 | 2,100 | 0.1 | 2,208 | 1,995 |

### 90% confidence intervals

Argon     $0.98 \, S^2 \leq \sigma_1^2 \leq 1.02 \, S^2$

$CO_2$     $0.95 \, S^2 \leq \sigma_1^2 \leq 1.05 \, S^2$

Therefore the argon variance has error bars of $\pm 2\%$ and $CO_2$ has $\pm 5\%$ for 90% confidence and 30-second record length.

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## 6.3  CORRELATION COEFFICIENT

The covariance of irradiance is defined as

$$C_I(\rho) \equiv \langle (I_1 - \bar{I}_1)(I_2 - \bar{I}_2) \rangle \tag{1}$$

where the angle brackets denote an ensemble average of all pairs of points in the receiving plane separated by a distance $\rho$. The variance of irradiance is simply the variance for zero separation

$$C_I(0) = \langle (I - \bar{I})^2 \rangle$$

In order to obtain a measure of the independence of the scintillation at two points in the receiving plane, the covariance is usually divided by the variance to obtain the irradiance-correlation coefficient

$$\gamma(\rho) = \frac{C_I(\rho)}{C_I(0)} = \frac{\langle (I_1 - \bar{I}_1)(I_2 - \bar{I}_2)\rangle}{\langle (I - \bar{I})^2 \rangle} \tag{2}$$

Fitzmaurice[1] has shown, that the irradiance-correlation coefficient may be related to log-amplitude statistics

$$\gamma(\rho) = \frac{\exp\left[4C_1(\rho)\right] - 1}{\exp\left[4C_1(0)\right] - 1} \tag{3}$$

where
$$C_1(\rho) = \langle (l_1 - \bar{I}_1)(l_2 - \bar{I}_2)\rangle \tag{4}$$

If we define the log-amplitude correlation coefficient as $F(\rho)$,

then
$$F(\rho) = \frac{C_1(\rho)}{C_1(0)}$$

$$\gamma(\rho) = \frac{\exp\left[4C_1(0)\,F(\rho)\right] - 1}{\exp\left[4C_1(0)\right] - 1} \tag{6}$$

For weak turbulence,

$$\gamma(\rho) = F(\rho) \tag{7}$$

From Fried[2],

$$C_1^s(\rho) = \frac{8.16 k^2 C_N^2 \Gamma(\frac{5}{6})}{4\pi} \quad \mathrm{Re}\left( \int_0^z \left[\left(\frac{s\rho}{z}\right)^{5/3} \Big/ \left[2^{8/3} \; \Gamma(11/6)\right]\right.\right.$$
$$\phantom{xxxxxxxxxxxxxxxxxxxxxxxxxxxxx} 5/6$$

$$-\frac{1}{2}\left[i\,s\,(z-s)/(kz)\right]^{5/6}\times\,_1F_1\left[-\frac{5}{6};1;\frac{ik\rho_s^{2}}{4\,(z-s)\,z}\right]\Bigg\}\,ds\Bigg) \tag{6}$$

For a propagation path over which $C_N^{2}=0$ for all $s$ except for a thin layer of turbulence of thickness $\Delta s<<s$ this may be replaced by

$$C_1^{s}(0)=\frac{8.16k^{2}C_N^{2}\,\Gamma(-5/6)\,\Delta s}{4\pi}\;\;\mathrm{Re}\left[\frac{\left(\frac{s\rho}{z}\right)^{5/3}}{2^{5/3}\,\Gamma\left(\frac{11}{6}\right)}-\frac{1}{2}\left\{\frac{is\,(z-s)}{kz}\right\}^{5/6}\right.$$

$$\times\,_1F_1\left[-\frac{5}{6};1;\frac{ik\rho^{2}s}{4\,(z-s)\,z}\right]\Bigg] \tag{9}$$

Furthermore the turbulence need not be constant and therefore $C_N^{2}\Delta s$ may be replaced by

$$\int_{\Delta s}C_N^{2}(s)ds \tag{10}$$

The log-amplitude correlation coefficient becomes

$$F(\rho)=\frac{\left[\left(\frac{s\rho}{z}\right)^{5/3}/\left[2^{5/3}\Gamma(11/6)\right]\right]-\frac{1}{2}\left[\frac{s\,(z-s)}{kz}\right]^{5/6}\mathrm{Re}\left\{i^{5/6}\,_1F_1\left[-\frac{5}{6};1;\frac{ik\rho^{2}s}{4(z-s)z}\right]\right\}}{-\frac{1}{2}\left[\frac{s\,(z-s)}{kz}\right]^{5/6}\;\;\mathrm{Re}\,(i^{5/6})}$$

$$=3.86\,\mathrm{Re}\left\{\right\}\;\;-\frac{\left(\frac{s\rho}{z}\right)^{5/3}}{\mathrm{Re}(i^{5/6})\,2^{5/3}\,\Gamma\,(11/6)}\;\;\left[\frac{kz}{s\,(z-s)}\right]^{5/6} \tag{11}$$

which reduces to

$$F(\rho)=3.86\,\mathrm{Re}\left\{\right\}-4.09\left[\frac{k\rho^{2}s}{4\,(z-s)\,z}\right]^{5/6} \tag{12}$$

The function $\,_1F_1\left[-\frac{5}{6};1;\frac{ik\rho^{2}s}{4(z-s)z}\right]$ is a confluent hypergeometric function with an imaginary argument.

6-24

Let

$$x = \frac{k_0{}^2 s}{4(z-s)z} = \left[\frac{\rho}{\sqrt{\frac{4(z-s)z}{ks}}}\right]^2 \equiv \left[\frac{\rho}{\sqrt{\lambda L}}\right]^2 \tag{13}$$

Then the hypergeometric function may be evaluated by a Kummers[3] function

$$_1F_1(a; b; ix) = 1 + \frac{a(ix)}{b} + \frac{(a)_2(ix)^2}{(b)_2\,2!} + \ldots \frac{(a)_n(ix)^n}{(b)_n\,n!} \tag{14}$$

where
$$(a)_n = a(a+1)(a+2)\ldots(a+n-1) \qquad (a)_0 = 1 \tag{15}$$

$$(b)_n = b(b+1)(b+2)\ldots(b+n-1) \qquad (b)_0 = 1 \tag{16}$$

If $a = -5/6$ and $b = 1$, then

$$_1F_1\left[-\frac{5}{6}; 1; ix\right] = 1 - 5/6(ix) + \frac{5}{144}x^2 + \frac{35}{7776}ix^3$$

$$+ \frac{91}{122,808}x^4 - \frac{1729}{186,624}ix^5 + \ldots$$

$$= 1 - 0.833\,ix + 0.0347x^2 + 0.00450\,ix^3 + 0.610x^4$$
$$- 0.0000772x^5 + \ldots \tag{17}$$

We may normalize equation 12 by replacing $\rho$ by $\rho/\sqrt{\lambda L}$, obtaining

$$F\left(\frac{\rho}{\sqrt{\lambda L}}\right) = 3.86\ \mathrm{Re}\{\} - 4.09\ [x]^{5/6} \tag{18}$$

Using this equation, we evaluate $\rho/\sqrt{\lambda L}$ , $\text{Re}\{\}$,

and $F\left(\dfrac{\rho}{\sqrt{\lambda L}}\right)$ as a function of x in the following:*

| x | $\dfrac{\rho}{\sqrt{\lambda L}}$ | $\text{Re}\left\{i^{5/6} \, {}_1F_1\left[-\frac{5}{6}:1;ix\right]\right\}$ | $F\left(\dfrac{\rho}{\sqrt{\lambda L}}\right)$ |
|---|---|---|---|
| 0 | 0. | 0.259 | 1.000 |
| 1/4 | 0.500 | 0.431 | 0.486 |
| 1/2 | 0.707 | 0.663 | 0.256 |
| 1 | 1.000 | 1.068 | 0.020 |
| 2 | 1.414 | 1.870 | -0.094 |

The log-amplitude correlation coefficient is shown as a function of the normalized parameter $\rho/\sqrt{\lambda L}$ in Figure 6-9. The normalizing factor is known as the correlation length and corresponds to the separation for which x = 1.

Using the relationship of equation 6 between $\gamma\left(\rho/\sqrt{\lambda L}\right)$ and $F\left(\rho/\sqrt{\lambda L}\right)$, we can calculate the theoretical dependence of irradiance-correlation coefficient:

|  | $\gamma\left(\rho/\sqrt{\lambda L}\right)$ | | |
|---|---|---|---|
| $F\left(\rho/\sqrt{\lambda L}\right)$ | $\sigma_1^2=0$ | $\sigma_1^2=0.20$ | $\sigma_1^2=0.40$ |
| 0 | 0 | 0 | 0 |
| 0.2 | 0.2 | 0.13 | 0.1 |
| 0.4 | 0.4 | 0.31 | 0.22 |
| 0.6 | 0.6 | 0.50 | 0.39 |
| 0.8 | 0.8 | 0.73 | 0.65 |
| 1.0 | 1.00 | 1.00 | 1.00 |

In order to evaluate the actual correlation coefficient, we must analyze the turbulence as though it is made up of thin layers of turbulence of varying strengths and at varying distances from the source.

Because the thermosonde measurement showed only one layer of turbulence and from other measurements we know that turbulence exists at the tropopause, we will use a two-layer model of the atmosphere, although the results of the analysis will make apparent how more complicated models can be handled.

*See more complete table in Appendix B

6-26

The y-axis is labeled LOG-AMPLITUDE CORRELATION COEFFICIENT $-F\left(\dfrac{\rho}{\sqrt{\lambda L}}\right)$ with values from $-0.2$ to $1.0$.

The x-axis is labeled $P/\sqrt{\lambda L}$ with values from 0.0 to 3.0.

8529-6.3-1
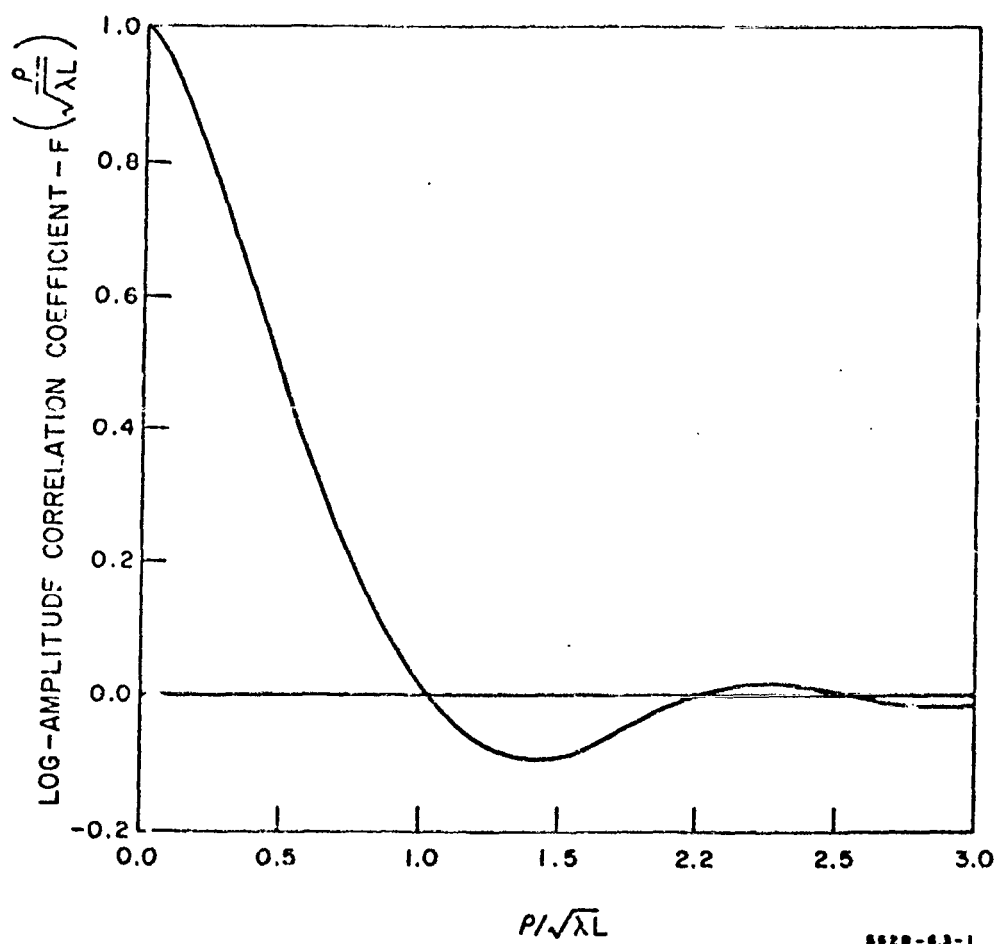
Figure 6-9.  Log-Amplitude Correlation Coefficient

For a single layer of turbulence,

$$C_1^s(\rho) = \frac{8.16\, k^2\, \Gamma(-5/6)}{4\pi} \quad C_N^2 \Delta s \ \ \text{Re}\left[\frac{\left(\frac{s\rho}{z}\right)^{5/3}}{2^{8/3}\Gamma(11/6)} - \frac{1}{2}\left\{\frac{sz}{kz}\right\}^{5/6} {}_1^{5/6}\right.$$

$$\left. \times \ {}_1F_1\left[-\frac{5}{6}; 1; \frac{ik\rho^2 s}{4(z-s)z}\right]\right] \tag{19}$$

(for $s << z$)

Let

$$A = \frac{8.16\, k^2\, \Gamma(-5/6)}{4\pi} \qquad B = \frac{(\rho/z)^{5/3}}{2^{8/3}\Gamma(11/6)}$$

$$C = \frac{1}{2}\left\{\frac{1}{k}\right\}^{5/6} \qquad D = \text{Re}\left\{i^{5/6}\,{}_1F_1[\ ]\right\}$$

Then

$$C_N^* = C_N^2 \Delta s$$

$$C_1^s(\rho) = AC_N^*\left[Bs^{5/3} - CDs^{5/6}\right]$$

Also

$$C_1^s(\omega) = -AC_N^*\, CDs^{5/6} \qquad D = 0.26 \tag{20}$$

$$= -0.26\, A\, C_N^*\, C\, s^{5/6} \qquad C_N^+ \quad C_N^* s^{5/6} = C_N^2 \Delta s\, s^{5/6}$$

$$= -0.26\, A\, C_N^+\, C$$

For a two layer turbulence model,

$$C_1^s(\rho) = AC_{N1}^*\left[Bs_1^{5/3} - CD_1 s^{5/6}\right] + AC_{N2}^*\left[Bs_2^{5/3} - CD_2 s_2^{5/6}\right] \tag{21}$$

If

$$s_1 << s_2 \qquad D_1 = 0.26$$

Therefore

$$C_1^s(\rho) = AC_{N1}^*\left[Bs_1^{5/3} - 0.26Cs_1^{5/6}\right] + AC_{N2}^*\left[Bs_2^{5/3} - CD_2 s_2^{5/6}\right] \tag{22}$$

$$= AC_{N1}^+\left[Bs_1^{5/6} - 0.26C\right] + AC_{N2}^+\left[Bs_2^{5/6} - CD_2\right]$$

Defining $F(\rho) = C_1^{\ s}(\rho)/C_1^{\ s}(o)$, we have for the single layer model

$$F(\rho_1)_1 = \frac{A C_N^+ \left[Bs_1^{5/6} - C\right]}{-0.26\ A C_N^*\ \ C} = \frac{CD_1 - Bs_1^{5/6}}{0.26\ C}$$

For the two layer model,

$$F(\rho)_{1+2} \quad \frac{A C_{N1}^+ \left[Bs_1^{5/6} - 0.26C\right] + AC_{N2}^+ \left[Bs_2^{5/6} - CD_2\right]}{-0.26\ AC\ \left[C_{N1}^+ + C_{N2}^+\right]}$$

$$= \frac{C_{N1}^+ \left[Bs_1^{5/6} - C D_1\right] - C_{N2}^+ \left[Bs_2^{5/6} - C D_2\right]}{-0.26\ C\ \left[C_{N1}^+ + C_{N2}^+\right]} \tag{23}$$

If $C_{N1}^+ \gg C_{N2}^+$

$$F(\rho)_{1+2} = \frac{C_{N1}^+ \left[C D_1 - Bs_1^{5/6}\right] + C_{N2}^+ \left[C D_2 - Bs_2^{5/6}\right]}{+0.26\ C\ C_{N1}^+}$$

$$= \frac{\left[C D_1 - Bs_1^{5/6}\right]}{0.26\ C} + \frac{C_{N2}^+}{C_{N1}^+} \frac{\left[C D_2 - Bs_2^{5/6}\right]}{0.26\ C}$$

$$= F(\rho)_1 + \frac{C_{N2}^+}{C_{N1}^+}\ F(\rho)_2 \tag{24}$$

If $C_{N1}^+ \approx C_{N2}^+$

$$F(\rho)_{1+2} = \left[\frac{C_{N1}^+}{C_{N1}^+ + C_{N2}^+}\right]\ F(\rho)_1 + \left[\frac{C_{N2}^+}{C_{N1}^+ + C_{N2}^+}\right] F(\rho)_2 \tag{25}$$

and for weak scintillation

$$\gamma_{1+2} = \left[\frac{C_{N1}^+}{C_{N1}^+ + C_{N2}^+}\right] \gamma_1 + \left[\frac{C_{N2}^+}{C_{N1}^+ + C_{N2}^+}\right] \gamma_2 \tag{26}$$

From this result, the method of analysis for multilayer models is apparent.

$$F(\rho) = \sum_{n=1}^{k} \left[\frac{C_{N1}^+}{C_{NT}^+}\right] F(\rho_1) + \left[\frac{C_{N2}^+}{C_{NT}^+}\right] F(\rho_2) + \ldots$$

$$\ldots \left[\frac{C_{Nk}^+}{C_{NT}^+}\right] F(\rho_k) \tag{27}$$

where $C_{NT}^+$ is the integrated value of $C_N^+$ from 0 to $\infty$

From Figure 6-10, the correlation length for a particular altitude can be obtained. Using this length, $\gamma_1$ and $\gamma_2$ can be calculated and, using the measured values of $C_{N1}^+$ and $C_{N2}^+$ from the experiment, a theoretical shape for $\Gamma(\rho)$ may be obtained. This has been done graphically in Figure 6-11 using the correlation coefficient curve of Figure 6-9 and the following paragraphs

| | h (m) | Argon $\sqrt{\lambda L}$ | $CO_2$ | $C_N^+/C_{NT}$ |
|---|---|---|---|---|
| Boundary Layer | 700 | 0.70 | 3..8 | 0.40 |
| Tropopause | 13,200 | 0.12 | 0.55 | 0.60 |

The experimental values of correlation coefficient are plotted in Figure 6-12 and recorded in Table 6-2. Using the results of Figure 6-12 and converting F($\rho$) to $\gamma$ through Table 6-2, the theoretical log-irradiance correlation coefficient for various values of log-amplitude variance are shown for comparison.

The experimental and theoretical fit show excellent agreement indicating that the theory appears to give an accurate prediction of correlation coefficient.
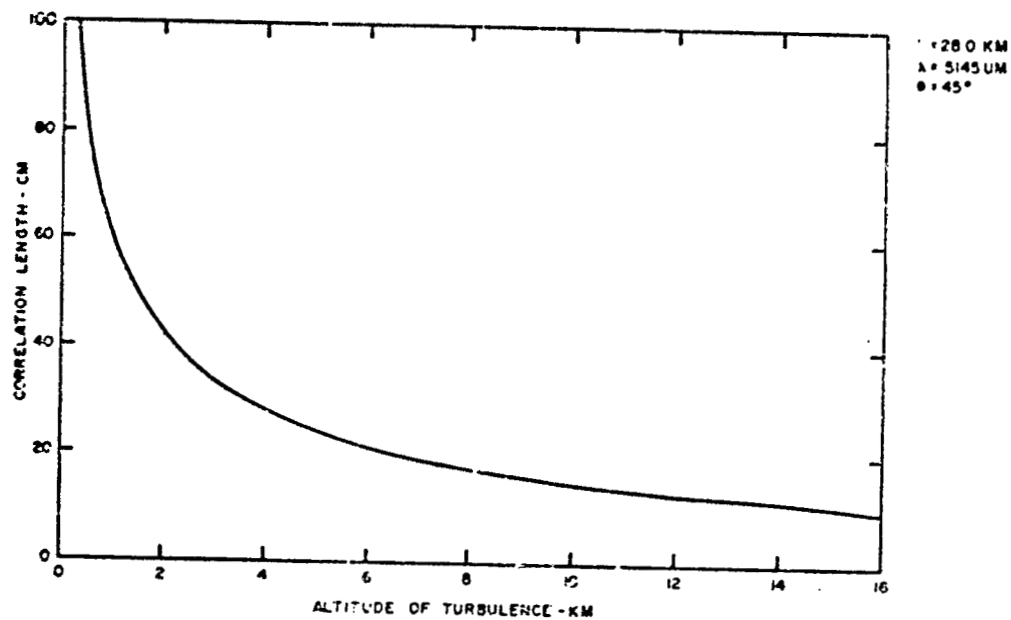
Figure 6-10. Correlation Length for Specific Altitude
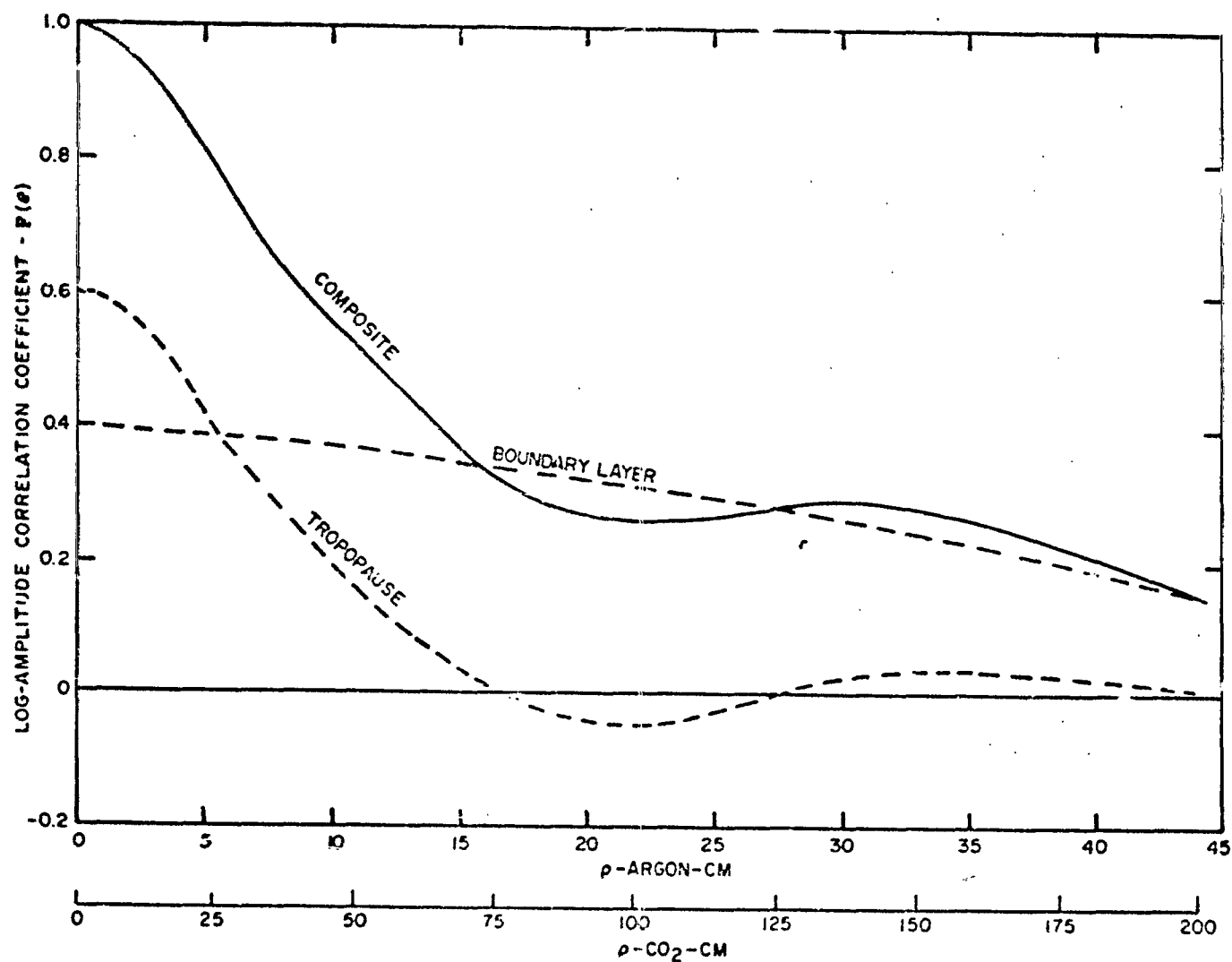
Figure 6-11. Theoretical Shape of F $(\rho)$
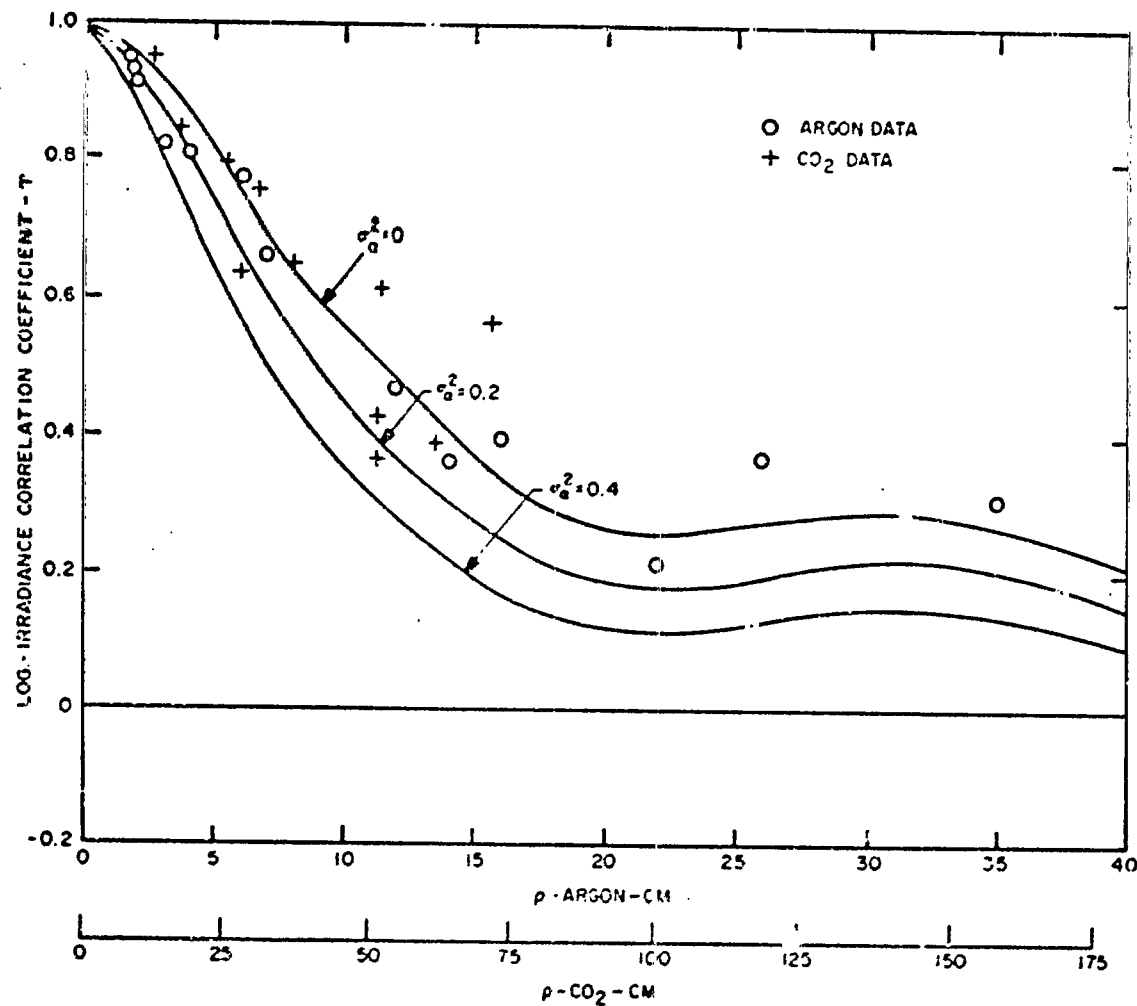
Figure 6-12. Experimental Values of Correlation Coefficient

Table 6-2
Table of Experimental Irradiance Correlation Coefficients

| Argon Data | | | | Carbon-Dioxide Data | | |
|---|---|---|---|---|---|---|
| | Time (GMT) | $\rho$(CM) | $\gamma$ | | Time (GMT) | $\rho$(CM) | $\gamma$ |
| 1. | 13 29 16 | 2 | 0.949 | 1. | 13 32 40 | 5 | 0.915 |
| 2. | 13 27 00 | 2 | 0.939 | 2. | 13 20 10 | 6 | 0.989 |
| 3. | .13 25 45 | 2 | 0.921 | 3. | 13 38 00 | 7 | 0.896 |
| 4. | 13 23 00 | 3 | 0.811 | 4. | 13 17 10 | 8 | 0.943 |
| 5. | 13 20 10 | 4 | 0.803 | 5. | 13 08 30 | 14 | 0.958 |
| 6. | 13 17 10 | 6 | 0.772 | 6. | 13 46 15 | 18 | 0.829 |
| 7. | 13 14 50 | 8 | 0.665 | 7. | 13 48 45 | 28 | 0.789 |
| 8. | 13 12 55 | 12 | 0.477 | 8. | 13 09 30 | 28 | 0.639 |
| 9. | 13 12 15 | 14 | 0.362 | 9. | 1 50 00 | 32 | 0.758 |
| 10. | 13 08 30 | 16 | 0.400 | 10. | 11 49 05 | 37 | 0.644 |
| 11. | 13 47 30 | 22 | 0.212 | 11. | 11 51 32 | 52 | 0.631 |
| 12. | 13 09 30 | 26 | 0.367 | 12. | 11 58 50 | 52 | 0.416 |
| 13. | 12 08 07 | 35 | 0.307 | 13. | 13 53 20 | 52 | 0.379 |
| | | | | 14. | 13 55 00 | 62 | 0.397 |
| | | | | 15. | 13 57 20 | 72 | 0.563 |

# REFERENCES

1. Fitzmaurice, M.W., Experimental Investigations of Optical Propagation in Atmospheric Turbulence, NASA TR R-370 (Aug 1971).

2. Fried, D.L. J. Optical Soc. Am., Vol. 57 (1967), p.181.

3. Abramowitz, M., and Stegun, I., Handbook of Mathematical Functions, Dover, New York (1964), p. 504.

## 6.4 APERTURE AVERAGING OF SCINTILLATION

Scintillation is a problem mainly to laser communications systems working in the visible or near infrared. Far-infrared wavelengths, because of the 7/6 power of wave-number dependence, are only slightly affected. There are only two methods to avoid scintillation. The first, selection of a site which has low turbulence, is partially effective, but since nearly half of the scintillation is due to tropopause which is too high to be affected by terrain changes, only small reductions can be made through site selection. Hulett[1] notes that astronomers, at many different locations around the world, observe nearly identical levels of scintillation.

The second method of reducing scintillation is through the use of large apertures to average out the intensity of the fluctuations. Since the intensity in the receiving plane, as has been shown in section 6.3, is not well correlated over separations larger than a few tens of cm, the aperture of a large receiver will be divided into many small cells which scintillate independently from adjacent cells. Because of this behavior, the scintillation as measured by

$$\sigma_p = \frac{\sqrt{\langle (P - \bar{P})^2 \rangle}}{\bar{P}} \tag{1}$$

for apertures containing many independent cells should be proportional to $1/\sqrt{n}$ where n is the number of independent areas. Also, if on the average all of these areas are the same size, the $\sqrt{n}$ is proportional to the diameter of the receiver (D) so that for large apertures $\sigma_p$ is proportional to $1/D$.

Figure 6-13 shows the aperture dependence of the average value of $\bar{\sigma}_p$ as measured by Bufton[2], Protheroe[3], and Young[4]. For very small apertures at wavelengths in the blue and green portion of the visible spectrum ($\lambda \approx 0.500 \pm 0.100 \, \mu m$) the magnitude of $\bar{\sigma}_p$ is approximately 0.25. As the aperture is increased, $\bar{\sigma}_p$ decreases rapidly until an aperture of about 25 cm (10 inches) is reached. Beyond 25 cm, the rate of decrease in scintillation is much slower, and approaches the theoretical $1/D$ dependence for large apertures. For $D \gtrsim 50$ cm (20 inches)

$$\bar{\sigma}_p \approx \frac{0.045}{D} \tag{2}$$
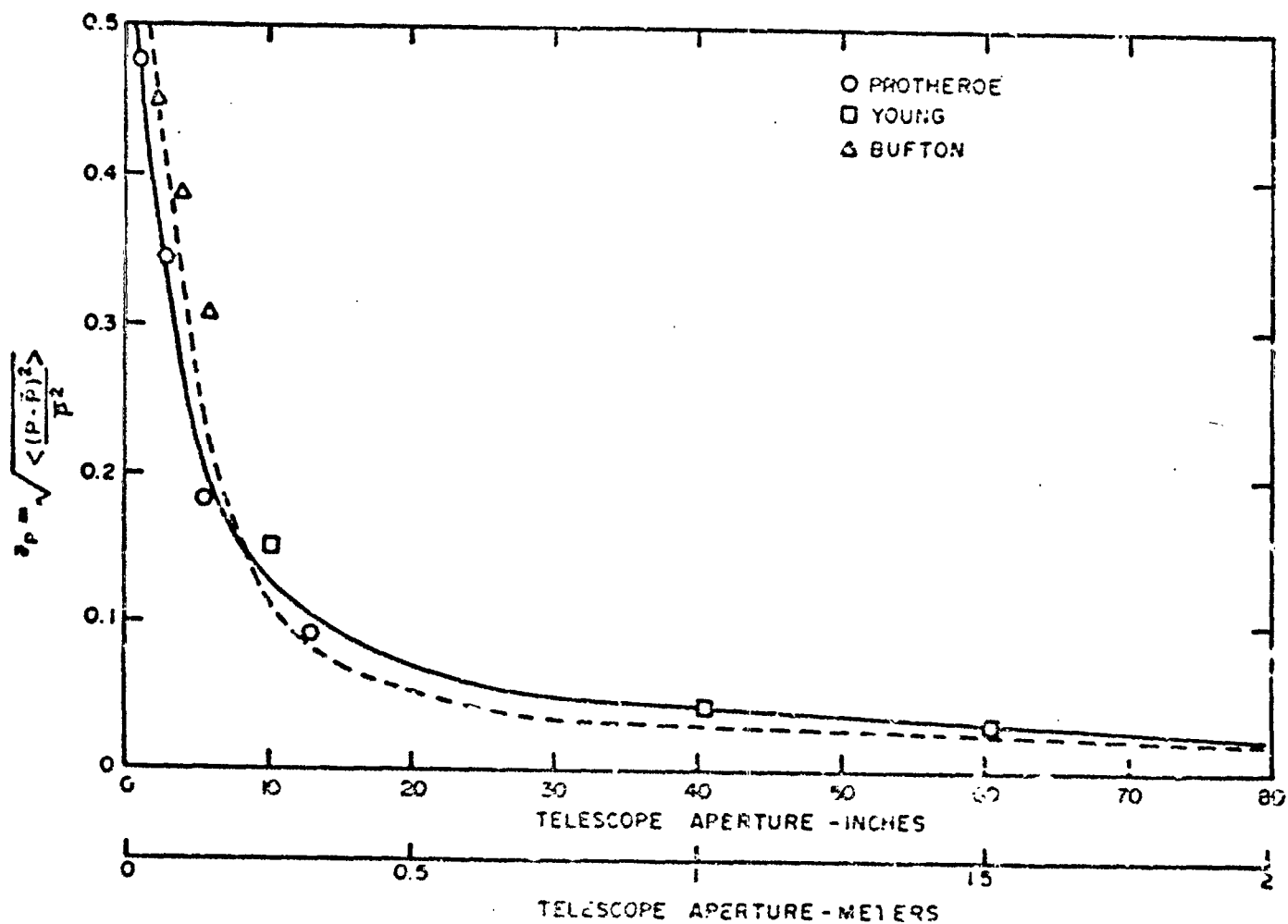
Figure 6-13. Aperture Dependence of $\bar{\sigma}_p$

The y-axis label reads: $\sigma_p = \sqrt{\dfrac{\langle (P-\bar{P})^2 \rangle}{\bar{P}^2}}$

Legend:
O PROTHEROE
□ YOUNG
△ BUFTON

X-axis (top): TELESCOPE APERTURE - INCHES
X-axis (bottom): TELESCOPE APERTURE - METERS

1052-6.4-1

## REFERENCES

1. Hulett, H.R., J. Optical Soc. Am., Vol. 57 (1967), p.1335.

2. Bufton, J.L., An Investigation of Atmospheric Turbulence by Stellar Observations, NASA TR R-369 (Aug 1971).

3. Protheroe, W.M., Preliminary Report on Stellar Scintillation, Contrib. Perkins Observ. Ser. 2, No. 447 (1955).

4. Young, J., Astronomical Journal (Aug 1967), P. 747.

## 6.5  AUTOCORRELATION OF IRRADIANCE AND POWER SPECTRAL DENSITY*

The autocorrelation of irradiance is defined as

$$R_I(\tau) = <[I(t)-\bar{I}] \, [I(t+\tau)-\bar{I}]> = C_I(\tau) \tag{1}$$

where $I(t)$ is the irradiance at time $t$, $I(t+\tau)$ is the irradiance at the same point at time $t+\tau$, and $\bar{I}$ is the average irradiance. This is usually normalized by dividing by the variance of irradiance to obtain the irradiance-correlation coefficient

$$\gamma(\tau) = \frac{C_I(\tau)}{C_I(0)} = \frac{<[I(t)-\bar{I}] \, [I(t+\tau)-\bar{I}]>}{<(I-\bar{I})^2>} \tag{2}$$

where the coefficient is now a function of time delay $\tau$ rather than of spatial displacement $(\rho)$ as in equation 2 of section 6.3. For a single layer of turbulence at distance s from the source, a displacement $\rho'$ corresponds to a displacement

$$\rho = \rho' \left[ \frac{z}{s} \right] \tag{3}$$

in the detector plane at z from the projection shown in Figure 6-14. If $\rho$ is replaced by the correlation length $\sqrt{\lambda L}$ in the detector plane,

$$\rho' = \sqrt{\lambda L} \left[ \frac{s}{z} \right] \tag{4}$$

The time necessary for a turbulon to traverse this distance is

$$\tau_0 = \frac{\sqrt{\lambda L}}{v_n} \left[ \frac{s}{z} \right] \tag{5}$$

where $v_n$ is the wind velocity normal to the propagation path.

Therefore if Taylor's hypothesis[1] of frozen-in turbulence is assumed, $\tau_0$ is the time required for a turbulon in the plane of turbulence at s to traverse the correlation length $(\sqrt{\lambda L})$ in the receiver plane, and is therefore simply the correlation time. Taylor's hypothesis is valid whenever $\sqrt{\lambda L} << L_0$, that is when the correlation length is less than the outer scale of turbulence. From Fried[2], the outer scale of turbulence

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*Data processed only for the argon wavelength.
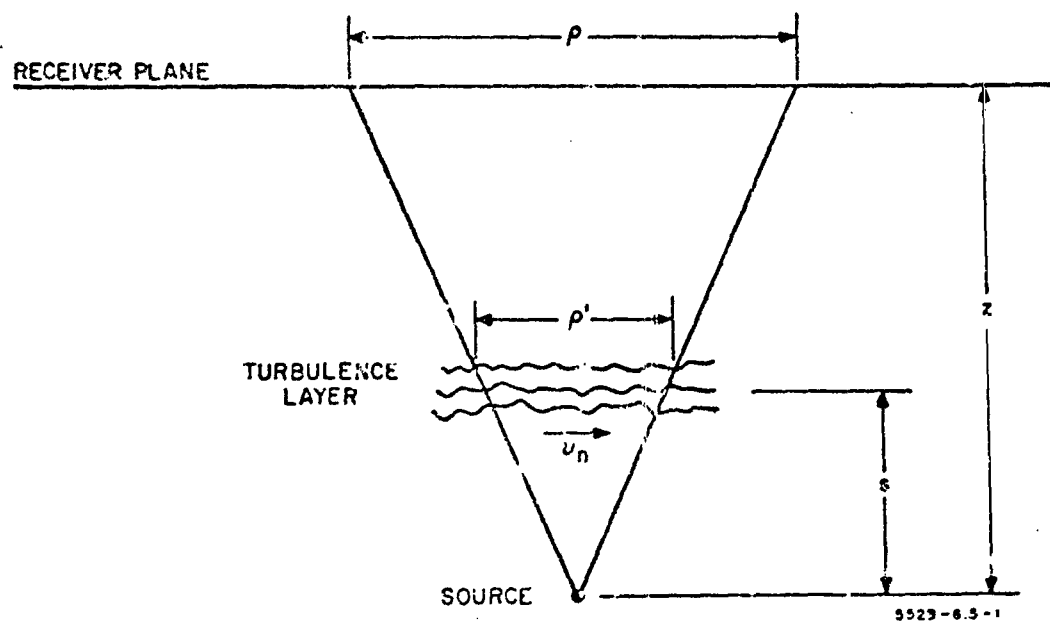
Figure 6-14

at 700 m altitude (lowest layer of turbulence) is 40 m which is much larger than the correlation length for that altitude (0.70 m) and therefore the requirements of Taylor's hypotehsis are met for all the turbulence layers since correlation length decreases with altitude while outer scale increases.

The wind speed corresponding to the tropopause (14.4 km) is 24.7 $ms^{-1}$ and the correlation length corresponding to this height is 0.087 meters. Therefore from equation 5, the correlation time is

$$\tau_o = \frac{0.087}{24.7} \left[ \frac{14.4 \times 10^3}{23.4 \times 10^3} \right] = 2.17 \times 10^{-3} \text{ sec}$$

For the low altitude barrier layer turbulence, we get the altitude of the turbulence directly from thermosonde measurements and find it to be approximately 700 meters, with a wind speed of 10.1 $ms^{-1}$ (19.6 knots). The correlation length for this altitude is approximately 0.70 meters, therefore

$$\tau_o = \frac{0.70}{10.1} \left[ \frac{0.70 \times 10^3}{23.4 \times 10^3} \right] = 2.08 \times 10^{-3} \text{ sec}$$

In making this computation the velocity of the wind has been used without correction for its azimuth since by fortuitous circumstance the wind vector for both layers of turbulence was precisely at right angles to the propagation path at the time of the measurement (1313 GMT). Had the wind been at another azimuth ($\phi_w$) the normal component of the wind velocity could have been computed by the following relationship from Young[4]

$$v_n = \frac{v}{\sec \Theta_T} \left[ 1 + \tan^2 \Theta_T \sin^2 (\phi_T - \phi_w) \right]^{1/2}$$

where $\Theta_T$ is the zenith angle of the target and $\phi_T$ is the azimuth angle. Since the barrier layer turbulence and tropopause turbulence were of approximately the same strength, we would expect the correlation time to be approximately

$$\tau_o = \frac{(2.08 + 2.17) \times 10^{-3}}{2} = 2.13 \times 10^{-3} \text{ sec.}$$

The actual correlation time from measurement (Figure 6-15) was $2.40 \times 10^{-3}$ seconds. The difference between predicted and measured correlation time is therefore

$$\frac{\tau_{oE} - \tau_{oP}}{\tau_{oE}} = \frac{(2.40 - 2.13) \times 10^{-3}}{2.40 \times 10^{-3}} = 11.2\%$$

which is well within the accuracy of the wind speed measurement.

The normalized power spectral density of irradiances calculated from the auto-correlation of irradiance data by Fourier transform methods is shown in Figure 6-16. Since the power spectral density of irradiance is proportional to wavenumber $(k = \frac{2\pi}{\lambda})$, by scaling the frequency axis, the same curve may be used for other wavelengths. Carbon dioxide scintillation for example would have frequencies

$$\sqrt{\frac{0.5145 \cdot 10^{-6}}{10.6 \times 10^{-6}}} = 0.22$$

times as great as the argon wavelength. The correlation curve can also be used for carbon dioxide by multiplying the time axis by $1/0.22$.

ARGON DATA
1343 Z

IRRADIANCE AUTO CORRELATION

TIME DELAY – T – MILLI SECONDS

Figure 6-15

6-43
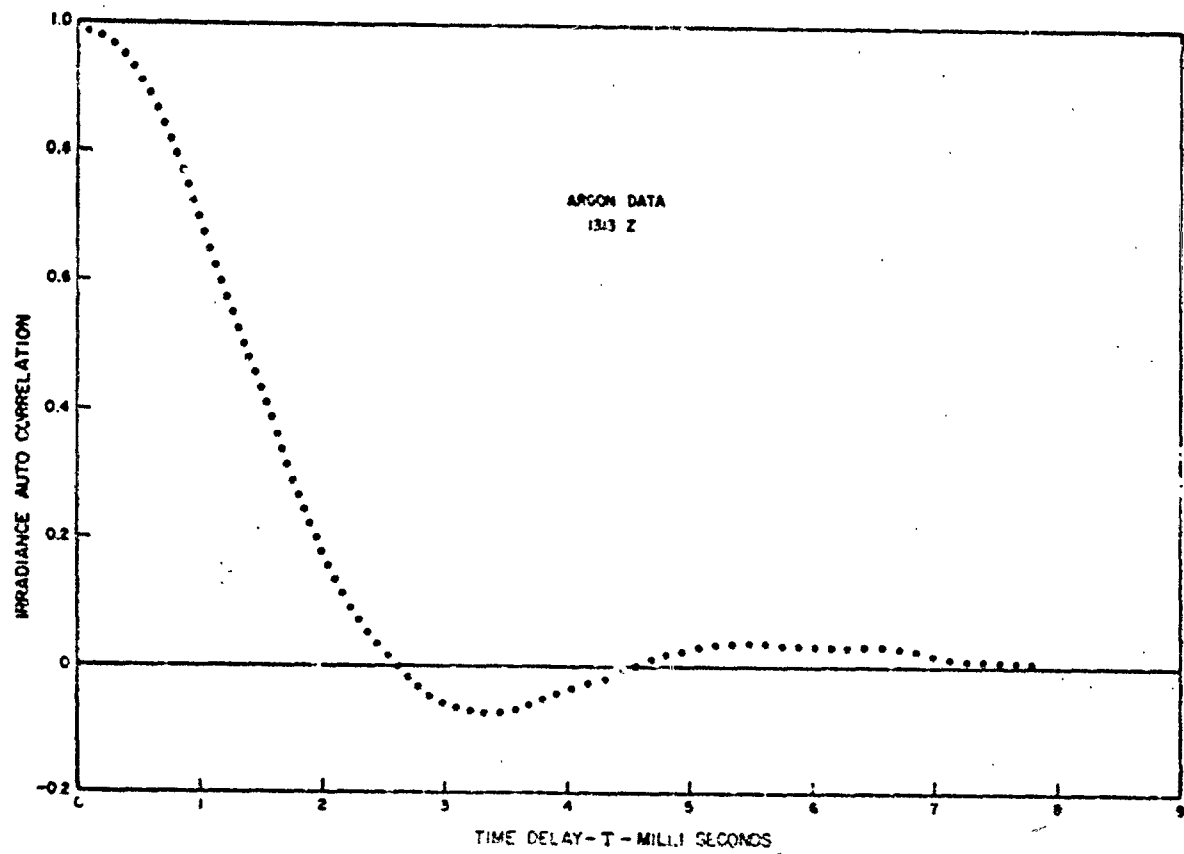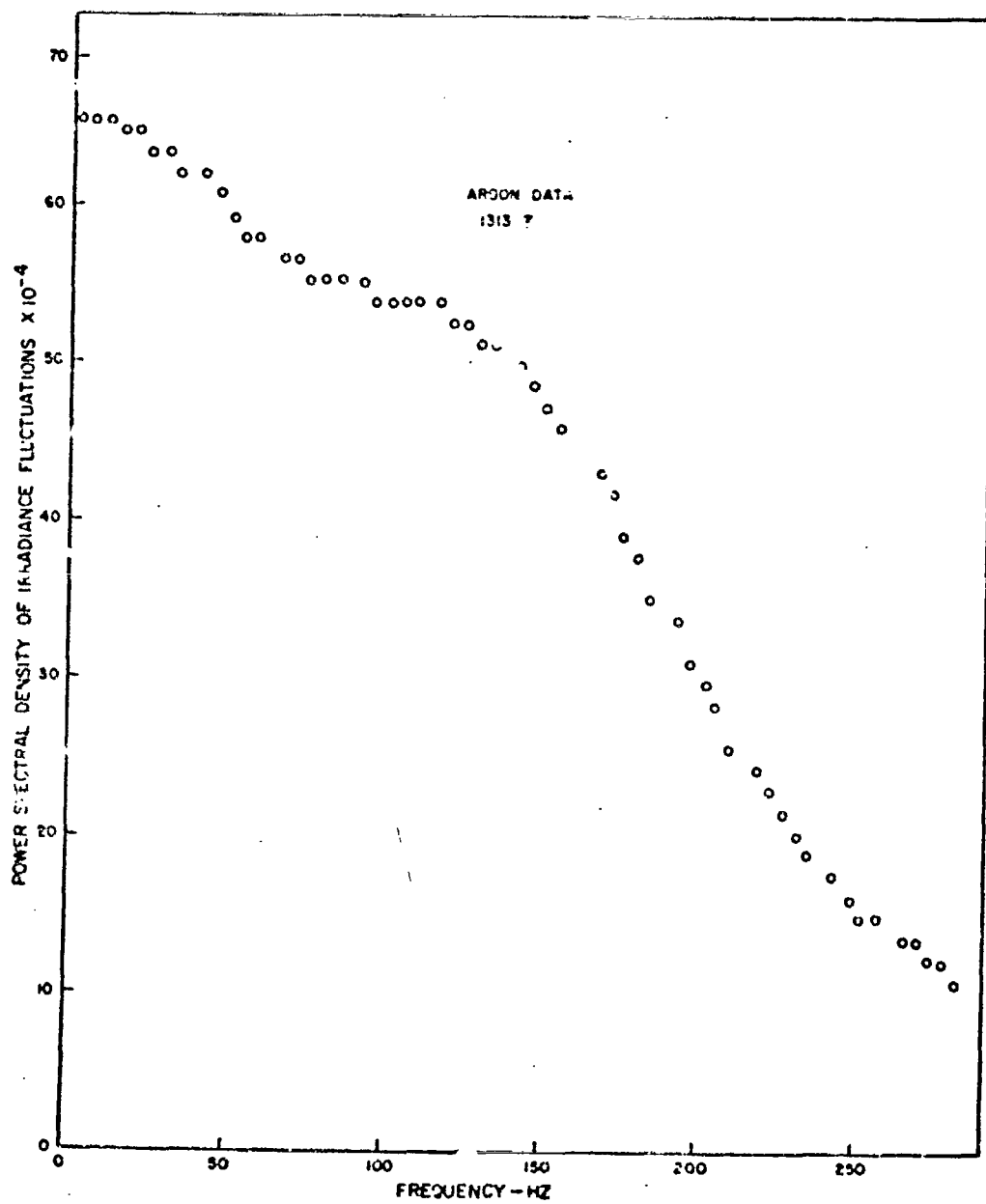
Figure 6-16

Table 6-3
Log-Irradiance Autocorrelation

| $\tau$ | $\gamma$ | $\tau$ | $\gamma$ | $\tau$ | $\gamma$ |
|---|---|---|---|---|---|
| 0 | 1.00 | 2.0 | 0.17 | 4.0 | -0.04 |
| 0.1 | 0.99 | 2.1 | 0.14 | 4.2 | -0.03 |
| 0.2 | 0.98 | 2.2 | 0.10 | 4.4 | -0.02 |
| 0.3 | 0.97 | 2.3 | 0.07 | 4.6 | 0 |
| 0.4 | 0.94 | 2.4 | 0.04 | 4.8 | 0.02 |
| 0.5 | 0.91 | 2.5 | 0.02 | 5.0 | 0.02 |
| 0.6 | 0.89 | 2.6 | 0 | 5.2 | 0.03 |
| 0.7 | 0.83 | 2.7 | -0.02 | 5.4 | 0.03 |
| 0.8 | 0.80 | 2.8 | -0.03 | 5.6 | 0.03 |
| 0.9 | 0.75 | 2.9 | -0.05 | 5.8 | 0.03 |
| 1.0 | 0.69 | 3.0 | -0.06 | 6.0 | 0.03 |
| 1.1 | 0.65 | 3.1 | -0.07 | 6.2 | 0.03 |
| 1.2 | 0.59 | 3.2 | -0.07 | 6.4 | 0.03 |
| 1.3 | 0.54 | 3.3 | -0.07 | 6.6 | 0.03 |
| 1.4 | 0.48 | 3.4 | -0.08 | 6.8 | 0.02 |
| 1.5 | 0.43 | 3.5 | -0.07 | 7.0 | 0.01 |
| 1.6 | 0.38 | 3.6 | -0.07 | 7.2 | 0.01 |
| 1.7 | 0.32 | 3.7 | -0.06 | 7.4 | 0.01 |
| 1.8 | 0.28 | 3.8 | -0.05 | 7.6 | 0.01 |
| 1.9 | 0.23 | 3.9 | -0.05 | 7.8 | 0 |

| Wavelength = 0.5145 µm | Time 13:12:55 | $\tau$ in milliseconds |
|---|---|---|

# REFERENCES

1.  Lumley, J.L., and Panofsky, H.A., The Structure of Atmospheric Turbulence, Interscience, New York (1964), p.74.

2.  Fried, D.L.  Proc IEEE, Vol. 55, No. 1 (Jan 1967), p. 57 (fig 5).

# 7. CONCLUSIONS

In the previous sections of this report we have described the experiment and analyzed its results. In this section we will attempt to synopsize what has been learned, and what can be concluded from the experiment results.

In section 6.1, Figure 6-4, the vertical profile of turbulence in the first 6 km of the atmosphere at the time of the experiment is presented, together with a mathematical model useful for approximating the turbulence distribution. We find that the scale height of the atmosphere is lower than predicted by Fried[1] by a factor of about three to one.

In section 6.2, the log-amplitude variance is analyzed, and it is shown that the strength of scintillation can only be accounted for by strong turbulence above 6 km. On the basis of the wind profile, temperature profile and correlation length, the layer is predicted to be at the tropopause. The distribution of log-amplitude variances is shown to be log-normal with a standard deviation of 0.6. The wavelength dependence $(k^{7/6})$ of scintillation is confirmed for the two wavelengths used in the test. The irradiance distribution is checked and found to be log-normal. The log-amplitude variance calculated on the basis of the profile of turbulence of section 6.1 is compared and found to be in agreement with the experimental values.

In section 6.3, the log-amplitude correlation coefficient is calculated on the basis of the turbulence model of section 6.1 and shown to be in agreement with the experimentally measured values. The correlation length is found to be proportional to $\sqrt{L}$, and a method of determining the correlation length for multilayer turbulence models is derived.

Based upon the results above, the reduction of scintillation using a large telescope is calculated in section 6.4 and compared to measurements by standard astronomical techniques. The dependence of scintillation frequency on wind speed is tested in section 6.5 and compared with the measured data. Correlation time and power spectral densities are shown to be directly related to wind speeds in the various turbulent layers.

The analysis of the data obtained from the experiment tends to confirm the theoretical predictions of scintillation based upon the turbulence profile. We believe that this is the first experiment in which the optical effects of turbulence upon laser beams

propagating over a vertical path have been tested in a controlled manner. We have found no case in which the data contradicts the theoretical predictions, and have obtained reasonably good confirmations of many of the aspects of the theory. The experiment also has allowed us to develop a technique of measuring the vertical profile through a balloon launched thermosonde device.

Our experiment has essentially tested Schmeltzer's Equation 7.8 for log-amplitude covariance. Because the assumptions made for the log-amplitude covariance equations are largely the same as those for phase covariance (Equation 7.9) the results of the experiment tend to lend credence to the theory for phase statistics as well as for amplitude statistics. The reader should review Schmeltzer's paper to see the remarkable similarity of Equations 7.8 and 7.9, and the fact that they are both based upon the same basic derivation.

Some of the more important results of the experiment are shown in the following table.

Table 7-1
Synopsis of Some of the Major Results of the HAPE I Experiment

| | |
|---|---|
| 1. Vertical Profile of Turbulence (Figure 6-4) | $C_N^2 = C_{No}^2 h^{-1/3} e^{-h/h_o} + \delta (h - h_p) C_{Np}^+$ |
| | $C_{No}^2 = 4 \times 10^{-14} m^{-2/3}$ $h_o = 1000 m$ |
| | $C_{Np}^+ = 4.3 \times 10^{-13} m^{1.3}$ $h_p = 13.2 \times 10^3 m$ |
| 2. Strength of Scintillation | |
| a. Wavelength dependence (p. 6-14) | $\alpha \lambda^{-7/6}$ |
| b. Distribution (Figures 6-8 and 6-9) | Log-normal |
| c. Log-Amplitude Variance (Mean) | |
| 1) 0.5145 μm (Figure 6-6) | 0.17 |
| 2) 10.6 μm (Figure 6-7) | 0.005 |
| d. Distribution of Sample Variances | |
| 1) Type (Figures 6-6 and 6-7) | Log-normal |
| 2) Standard Deviation (p. 6-14) | 0.6 |

Table 7-1
Synopsis of Some of the Major Results of the BAPE 1 Experiment (Cont)

3. Correlation of Intensity

   a. Wavelength Dependence of Correlation Length (Figure 6-13)     $\lambda^{1/2}$

   b. Correlation Lengths

      1) 0.5145 μm (Figure 6-13)     22 cm

      2) 10.6 μm (Figure 6-13)     100 cm

   c. Height Dependence of Correlation Length (p. 6-26)     $\alpha \left[ \dfrac{(z-s)z}{s} \right]^{1/2}$

4. Aperture Averaging of Scintillation

   a. Wavelength Dependence (p. 6-36)     $\alpha \lambda^{-1/2}$

   b. Aperture Dependence (Large Apertures) (Figure 6-14)     $\alpha D^{-1}$

   c. Standard Deviation of Power (Large Apertures) (p. 6-36)     $\alpha \sigma_p = \dfrac{0.045}{D}$

5. Correlation Time

   a. Wind Speed Dependence (p. 6-44)     $\alpha v_n^{-1}$

   b. Distance Dependence (p. 6-44)     $\alpha s/z$

   c. Wavelength Dependence (p. 6-44)     $\alpha \lambda^{1/2}$

Let us now consider how the results of the experiment have aided laser communication theory. From the results, we can estimate the time dependence of power received at a ground receiver from a spacecraft, or the power received at a spacecraft from a ground receiver. We can do this for any wavelength, for any size receiver, for any zenith angle, and (using results from BAPE II) for any time of day. This information can be directly applied to the estimation of bit error rate. We can estimate the length of severe dropouts, their frequency, and the percent of time that a laser communications system will operate at its designed bit error rate or signal-to-noise ratio. Because of the multitude of factors affecting the system performance, it is very difficult to present the results in a single graph. Rather, the results must be used to compute system performance for each new system as it is developed. Our new knowledge of the

atmospheric effects of turbulence should enable us to design laser systems which will operate successfully through atmospheric turbulence.

One area which needs further investigation is the frequency and depth of severe intensity dropouts. These dropouts have little effect upon the means and variances but severely effect the bit error rate of pulsed binary laser communications systems. Because of the very high signal-to-noise ratio and dynamic range required to perform this type of measurement, it was beyond the scope of this experiment.

A continuation of the BAPE I flight of 1970 was performed in the fall of 1971. This experiment, BAPE II, has added to the data of this report (particularly in the area of turbulence profiles) and a report on its results is under preparation.

The BAPE system is the only existing equipment capable of measuring the statistics of the optical effects of turbulence upon laser beams propagating from earth to space. While the BAPE I and II flights have provided a great deal of information, many interesting and important experiments remain to be done. The most important of these are the direct measurement of the phase statistics of optical propagation, and the measurement of the frequency of severe amplitude dropouts. Work in these areas should be the next problems of laser propagation to be investigated.

APPENDIX A
UPPER AIR DATA

UPPER AIR DATA

STATION ALTITUDE 4690.00 FEET MSL
21 OCT. 78    0300 HRS MST
ASCENSION NO. 174

GESTR SITE COORDINATES
575700.00 FEET E
352000.00 FEET N

| GEOMETRIC ALTITUDE MSL FEET | PRESSURE MILLIBARS | TEMPERATURE AIR DEGREES CENTIGRADE | DEW POINT DEGREES CENTIGRADE | REL. HUM. PERCENT | DENSITY GM/CUBIC METER | SPEED OF SOUND KNOTS | WIND DATA DIRECTION DEGREES(TN) | SPEED KNOTS | INDEX OF REFRACTION |
|---|---|---|---|---|---|---|---|---|---|

UPPER AIR DATA

STATION ALTITUDE 4690.00 FEET MSL
21 OCT. 78    0300 HRS MST
ASCENSION NO. 174

GESTR SITE COORDINATES
575700.00 FEET E
352000.00 FEET N

| GEOMETRIC ALTITUDE MSL FEET | PRESSURE MILLIBARS | TEMPERATURE AIR DEGREES CENTIGRADE | DEW POINT DEGREES CENTIGRADE | REL. HUM. PERCENT | DENSITY GM/CUBIC METER | SPEED OF SOUND KNOTS | WIND DATA DIRECTION DEGREES(TN) | SPEED KNOTS | INDEX OF REFRACTION |
|---|---|---|---|---|---|---|---|---|---|

** AT LEAST ONE ASSUMED RELATIVE HUMIDITY VALUE WAS USED IN THE INTERPOLATION.

UPPER AIR DATA

STATION ALTITUDE 4690.00 FEET MSL
21 OCT. 78    0300 HRS MST
ASCENSION NO. 174

GESTR SITE COORDINATES
575700.00 FEET E
352000.00 FEET N

| GEOMETRIC ALTITUDE MSL FEET | PRESSURE MILLIBARS | TEMPERATURE AIR DEGREES CENTIGRADE | DEW POINT DEGREES CENTIGRADE | REL. HUM. PERCENT | DENSITY GM/CUBIC METER | SPEED OF SOUND KNOTS | WIND DATA DIRECTION DEGREES(TN) | SPEED KNOTS | INDEX OF REFRACTION |
|---|---|---|---|---|---|---|---|---|---|

UPPER AIR DATA

STATION ALTITUDE 4690.00 FEET MSL
21 OCT. 78    0300 HRS MST
ASCENSION NO. 174

GESTR SITE COORDINATES
575700.00 FEET E
352000.00 FEET N

| GEOMETRIC ALTITUDE MSL FEET | PRESSURE MILLIBARS | TEMPERATURE AIR DEGREES CENTIGRADE | DEW POINT DEGREES CENTIGRADE | REL. HUM. PERCENT | DENSITY GM/CUBIC METER | SPEED OF SOUND KNOTS | WIND DATA DIRECTION DEGREES(TN) | SPEED KNOTS | INDEX OF REFRACTION |
|---|---|---|---|---|---|---|---|---|---|

** AT LEAST ONE ASSUMED RELATIVE HUMIDITY VALUE WAS USED IN THE INTERPOLATION.

STATION ALTITUDE 4050.00 FEET MSL          UPPER AIR DATA                    MSTR SITE COORDINATES
21 OCT. 70        0303 HRS PST              2940010734                       575700.00 FEET Y
ASCENSION NO. 334                           MCCLELLAN                        934000.00 FEET X

| GEOPOTENTIAL ALTITUDE MSL FEET | PRESSURE MILLIBARS | TEMPERATURE AIR DEGREES CENTIGRADE | DEWPOINT DEGREES CENTIGRADE | RELATIVE HUMIDITY PERCENT | DENSITY GM/CUBIC METER | SPEED OF SOUND KNOTS | WIND DATA DIRECTION DEGREES/TRUE | SPEED KNOTS | INDEX OF REFRACTION |
|---|---|---|---|---|---|---|---|---|---|
| 44000.0 | 43.0 | -55.2 | | | 98.9 | 570.0 | .340 | 6.7 | 1.000033 |
| 44200.0 | 42.4 | -56.6 | | | 98.0 | 570.4 | 290.4 | 7.0 | 1.000022 |
| 44400.0 | 41.9 | -56.3 | | | 97.1 | 573.3 | 260.4 | 8.4 | 1.000022 |
| 44600.0 | 41.3 | -56.4 | | | 96.2 | 570.6 | 292.0 | 10.0 | 1.000021 |
| 44800.0 | 40.7 | -56.3 | | | 95.2 | 570.7 | | | 1.000021 |
| 42000.0 | 40.1 | -56.1 | | | 94.3 | 570.0 | | | .000021 |
| 45400.0 | 30.5 | -56.1 | | | 93.3 | 570.9 | | | .000021 |
| 45600.0 | 31.3 | -56.0 | | | 92.4 | 571.0 | | | 1.000021 |
| 45800.0 | 37.4 | -57.3 | | | 91.5 | 571.2 | | | 1.000020 |
| 46000.0 | 36.4 | -57.4 | | | 90.7 | 571.4 | | | 1.000020 |
| 46200.0 | 35.9 | -57.4 | | | 89.8 | 571.5 | | | 1.000020 |
| 46400.0 | 35.3 | -57.5 | | | 88.0 | 571.5 | | | 1.000020 |
| 46600.0 | 34.6 | -57.5 | | | 87.2 | 571.6 | | | 1.000019 |

STATION ALTITUDE 4050.00 FEET MSL          MANDATORY LEVELS                 MSTR SITE COORDINATES
21 OCT. 70        0303 HRS PST              2940010734                       575700.00 FEET Y
ASCENSION NO. 334                           MCCLELLAN                        934000.00 FEET X

| PRESSURE MILLIBARS | GEOPOTENTIAL FEET | TEMPERATURE AIR DEGREES CENTIGRADE | DEWPOINT DEGREES CENTIGRADE | RELATIVE HUMIDITY PERCENT | WIND DATA DIRECTION DEGREES/TRUE | SPEED KNOTS |
|---|---|---|---|---|---|---|
| 850.0 | 4579. | 18.0 | 7.3 | 60. | 217.1 | 11.3 |
| 800.0 | 6172. | 13.6 | 1.3 | 49. | 261.9 | 20.3 |
| 750.0 | 8459. | 9.9 | -5.3 | 42. | 275.9 | 25.4 |
| 700.0 | 10242. | 6.7 | -17.3 | 19. | 275.5 | 24.8 |
| 650.0 | 12232. | 1.6 | -7.3 | 19. | 267.3 | 10.0 |
| 600.0 | 14324. | -3.2 | -24.7 | 17. | 255.7 | 31.1 |
| 550.0 | 16549. | -9.6 | -29.7 | 18. | 259.5 | 20.1 |
| 500.0 | 18951. | -17.3 | -30.4 | 22. | 263.5 | 30.4 |
| 450.0 | 21573. | -24.2 | -14.3 | 19. | 265.0 | 39.5 |
| 400.0 | 24473. | -25.2 | -41.4 | 18. | 269.5 | 39.7 |
| 350.0 | 27747. | -30.8 | -47.5 | 18. | 266.4 | 35.3 |
| 300.0 | 31117. | -39.9 | -50.4 | 12.** | 261.3 | 31.2 |
| 250.0 | 35154. | -50.0 | | | 270.3 | 45.3 |
| 200.0 | 34732. | -54.5 | | | 171.0 | 66.0 |
| 150.0 | 42713. | -61.4 | | | 274.0 | 18.3 |
| 125.0 | 46403. | -57.1 | | | 274.0 | 30.2 |
| 100.0 | 51153. | -57.5 | | | 271.3 | 45.3 |
| 30.0 | 53499. | -59.2 | | | 265.7 | 36.7 |
| 50.0 | 58219. | -55.2 | | | 244.7 | 21.3 |
| 30.0 | 66492. | -55.9 | | | 256.5 | 15.5 |
| 40.0 | 64195. | -58.5 | | | 254.7 | 9.4 |

** AT LEAST ONE ESTIMATED RELATIVE HUMIDITY VALUE WAS USED IN THE INTERPOLATION.

APPENDIX B

EVALUATION OF LOG-AMPLITUDE CORRELATION COEFFICIENT

## EVALUATION OF LOG-AMPLITUDE CORRELATION COEFFICIENT

COEFFICIENTS OF THE KUMMERS FUNCTION ARE:

```
0.10000000D 01    -.83333333D C0    -.34722222D-01
-.43010288D-02    -.60951432(-03    -.77205147D-04
-.89357809)-05    -.94220819D-06    -.9)7350050-07
-.803247830-08    -.65598573D-09    -.49695884D-10
-.35086218D-11    -.23183201D-12    -.14133330-13
-.84213605D-15    -.46602581D-16    -.24456949D-17
-.12203311D-18    -.58030527D-20    -.23155J10-21
```

THE A1, A2 COEFFICIENT VALUES ARE:

```
0.38637033D 01       0.41074568D 01
```

| X | P/LL | RE | F |
|---|---|---|---|
| 0.0 | 0.0 | 0.259 | 1.003 |
| 0.250 | 0.500 | 0.461 | 0.480 |
| 0.500 | 0.707 | 0.663 | 0.256 |
| 1.000 | 1.000 | 1.068 | 0.020 |
| 2.000 | 1.414 | 1.870 | -0.014 |

| X | P/LL | RE | F | X | P/LL | RE | F |
|---|---|---|---|---|---|---|---|
| 0.0 | 0.0 | 0.259 | 1.003 | 0.058 | 0.240 | 0.305 | 0.793 |
| 0.000 | 0.010 | 0.259 | 0.993 | 0.063 | 0.250 | 0.309 | 0.787 |
| 0.000 | 0.020 | 0.259 | 0.995 | 0.068 | 0.260 | 0.313 | 0.775 |
| 0.001 | 0.030 | 0.260 | 0.991 | 0.073 | 0.270 | 0.318 | 0.764 |
| 0.002 | 0.040 | 0.260 | 0.985 | 0.078 | 0.280 | 0.322 | 0.752 |
| 0.003 | 0.050 | 0.261 | 0.950 | 0.084 | 0.290 | 0.327 | 0.743 |
| 0.004 | 0.060 | 0.262 | 0.973 | 0.090 | 0.300 | 0.331 | 0.723 |
| 0.005 | 0.070 | 0.263 | 0.966 | 0.096 | 0.310 | 0.336 | 0.716 |
| 0.006 | 0.080 | 0.264 | 0.957 | 0.102 | 0.320 | 0.341 | 0.704 |
| 0.008 | 0.090 | 0.265 | 0.951 | 0.109 | 0.330 | 0.347 | 0.692 |
| 0.010 | 0.100 | 0.267 | 0.943 | 0.116 | 0.340 | 0.352 | 0.680 |
| 0.012 | 0.110 | 0.269 | 0.934 | 0.123 | 0.350 | 0.358 | 0.667 |
| 0.014 | 0.120 | 0.270 | 0.923 | 0.130 | 0.360 | 0.363 | 0.655 |
| 0.017 | 0.130 | 0.272 | 0.916 | 0.137 | 0.370 | 0.369 | 0.643 |
| 0.020 | 0.140 | 0.275 | 0.903 | 0.144 | 0.380 | 0.375 | 0.631 |
| 0.023 | 0.150 | 0.277 | 0.896 | 0.152 | 0.390 | 0.381 | 0.619 |
| 0.026 | 0.160 | 0.279 | 0.886 | 0.160 | 0.400 | 0.388 | 0.606 |
| 0.029 | 0.170 | 0.282 | 0.876 | 0.168 | 0.410 | 0.394 | 0.594 |
| 0.032 | 0.180 | 0.285 | 0.865 | 0.176 | 0.420 | 0.401 | 0.582 |
| 0.036 | 0.190 | 0.288 | 0.854 | 0.185 | 0.430 | 0.408 | 0.573 |
| 0.040 | 0.200 | 0.291 | 0.844 | 0.194 | 0.440 | 0.415 | 0.558 |
| 0.044 | 0.210 | 0.294 | 0.832 | 0.203 | 0.450 | 0.422 | 0.546 |
| 0.048 | 0.220 | 0.298 | 0.821 | 0.212 | 0.460 | 0.430 | 0.534 |
| 0.053 | 0.230 | 0.301 | 0.810 | 0.221 | 0.470 | 0.437 | 0.522 |

| X | P/LL | RE | F | X | P/LL | RE | F |
|---|------|-----|-----|---|------|-----|-----|
| 0.230 | 0.480 | 0.445 | 0.513 | 1.254 | 1.120 | 1.274 | -0.019 |
| 0.240 | 0.490 | 0.453 | 0.499 | 1.277 | 1.130 | 1.292 | -0.043 |
| 0.250 | 0.500 | 0.461 | 0.486 | 1.300 | 1.140 | 1.310 | -0.047 |
| 0.260 | 0.510 | 0.469 | 0.474 | 1.323 | 1.150 | 1.329 | -0.051 |
| 0.270 | 0.520 | 0.477 | 0.462 | 1.346 | 1.150 | 1.347 | -0.054 |
| 0.281 | 0.530 | 0.486 | 0.450 | 1.369 | 1.170 | 1.366 | -0.057 |
| 0.292 | 0.540 | 0.494 | 0.439 | 1.392 | 1.180 | 1.385 | -0.061 |
| 0.303 | 0.550 | 0.503 | 0.427 | 1.416 | 1.190 | 1.404 | -0.064 |
| 0.314 | 0.560 | 0.512 | 0.415 | 1.440 | 1.200 | 1.423 | -0.066 |
| 0.325 | 0.570 | 0.521 | 0.404 | 1.464 | 1.210 | 1.443 | -0.069 |
| 0.336 | 0.580 | 0.530 | 0.393 | 1.488 | 1.220 | 1.462 | -0.072 |
| 0.348 | 0.590 | 0.540 | 0.381 | 1.513 | 1.230 | 1.492 | -0.074 |
| 0.360 | 0.600 | 0.550 | 0.370 | 1.538 | 1.240 | 1.502 | -0.076 |
| 0.372 | 0.610 | 0.559 | 0.359 | 1.563 | 1.250 | 1.522 | -0.079 |
| 0.384 | 0.620 | 0.569 | 0.348 | 1.588 | 1.260 | 1.542 | -0.081 |
| 0.397 | 0.630 | 0.579 | 0.337 | 1.613 | 1.270 | 1.562 | -0.082 |
| 0.410 | 0.640 | 0.590 | 0.326 | 1.638 | 1.280 | 1.582 | -0.084 |
| 0.423 | 0.650 | 0.600 | 0.316 | 1.664 | 1.290 | 1.603 | -0.085 |
| 0.436 | 0.660 | 0.611 | 0.305 | 1.690 | 1.300 | 1.624 | -0.087 |
| 0.449 | 0.670 | 0.622 | 0.294 | 1.716 | 1.310 | 1.644 | -0.088 |
| 0.462 | 0.680 | 0.633 | 0.284 | 1.742 | 1.320 | 1.665 | -0.090 |
| 0.476 | 0.690 | 0.644 | 0.274 | 1.769 | 1.330 | 1.686 | -0.091 |
| 0.490 | 0.700 | 0.655 | 0.264 | 1.796 | 1.340 | 1.708 | -0.092 |
| 0.504 | 0.710 | 0.666 | 0.253 | 1.823 | 1.350 | 1.729 | -0.092 |
| 0.518 | 0.720 | 0.678 | 0.243 | 1.850 | 1.360 | 1.751 | -0.093 |
| 0.533 | 0.730 | 0.690 | 0.234 | 1.877 | 1.370 | 1.772 | -0.094 |
| 0.548 | 0.740 | 0.702 | 0.224 | 1.904 | 1.380 | 1.794 | -0.094 |
| 0.563 | 0.750 | 0.714 | 0.214 | 1.932 | 1.390 | 1.816 | -0.094 |
| 0.578 | 0.760 | 0.726 | 0.205 | 1.960 | 1.400 | 1.833 | -0.094 |
| 0.593 | 0.770 | 0.738 | 0.196 | 1.988 | 1.410 | 1.860 | -0.094 |
| 0.608 | 0.780 | 0.751 | 0.186 | 2.016 | 1.420 | 1.883 | -0.094 |
| 0.624 | 0.790 | 0.764 | 0.177 | 2.045 | 1.430 | 1.905 | -0.094 |
| 0.640 | 0.800 | 0.777 | 0.169 | 2.074 | 1.440 | 1.928 | -0.094 |
| 0.656 | 0.810 | 0.790 | 0.160 | 2.103 | 1.450 | 1.951 | -0.094 |
| 0.672 | 0.820 | 0.803 | 0.151 | 2.132 | 1.460 | 1.973 | -0.093 |
| 0.689 | 0.830 | 0.816 | 0.142 | 2.161 | 1.470 | 1.996 | -0.093 |
| 0.706 | 0.840 | 0.830 | 0.134 | 2.190 | 1.480 | 2.020 | -0.092 |
| 0.723 | 0.850 | 0.843 | 0.126 | 2.220 | 1.490 | 2.043 | -0.091 |
| 0.740 | 0.860 | 0.857 | 0.118 | 2.250 | 1.500 | 2.066 | -0.090 |
| 0.757 | 0.870 | 0.871 | 0.113 | 2.280 | 1.510 | 2.090 | -0.090 |
| 0.774 | 0.880 | 0.885 | 0.102 | 2.310 | 1.520 | 2.113 | -0.088 |
| 0.792 | 0.890 | 0.900 | 0.094 | 2.341 | 1.530 | 2.137 | -0.087 |
| 0.810 | 0.900 | 0.914 | 0.087 | 2.372 | 1.540 | 2.161 | -0.086 |
| 0.828 | 0.910 | 0.929 | 0.080 | 2.403 | 1.550 | 2.185 | -0.085 |
| 0.846 | 0.920 | 0.944 | 0.072 | 2.434 | 1.560 | 2.209 | -0.083 |
| 0.865 | 0.930 | 0.959 | 0.065 | 2.465 | 1.570 | 2.233 | -0.002 |
| 0.884 | 0.940 | 0.974 | 0.058 | 2.496 | 1.580 | 2.258 | -0.081 |
| 0.903 | 0.950 | 0.989 | 0.052 | 2.528 | 1.590 | 2.282 | -0.079 |
| 0.922 | 0.960 | 1.005 | 0.045 | 2.560 | 1.600 | 2.307 | -0.077 |
| 0.941 | 0.970 | 1.020 | 0.039 | 2.592 | 1.610 | 2.332 | -0.076 |
| 0.960 | 0.980 | 1.036 | 0.032 | 2.624 | 1.620 | 2.356 | -0.074 |
| 0.980 | 0.990 | 1.052 | 0.026 | 2.657 | 1.630 | 2.381 | -0.072 |
| 1.000 | 1.000 | 1.068 | 0.020 | 2.690 | 1.640 | 2.406 | -0.073 |
| 1.020 | 1.010 | 1.085 | 0.014 | 2.723 | 1.650 | 2.432 | -0.069 |
| 1.040 | 1.020 | 1.101 | 0.009 | 2.756 | 1.660 | 2.457 | -0.067 |
| 1.061 | 1.030 | 1.118 | 0.003 | 2.789 | 1.670 | 2.482 | -0.065 |
| 1.092 | 1.040 | 1.134 | -0.002 | 2.822 | 1.680 | 2.508 | -0.063 |
| 1.103 | 1.050 | 1.151 | -0.007 | 2.856 | 1.690 | 2.533 | -0.061 |
| 1.124 | 1.060 | 1.168 | -0.012 | 2.890 | 1.700 | 2.559 | -0.059 |
| 1.145 | 1.070 | 1.186 | -0.017 | 2.924 | 1.710 | 2.585 | -0.057 |
| 1.166 | 1.080 | 1.203 | -0.022 | 2.958 | 1.720 | 2.611 | -0.055 |
| 1.188 | 1.090 | 1.220 | -0.027 | 2.993 | 1.730 | 2.637 | -0.053 |
| 1.210 | 1.100 | 1.238 | -0.031 | 3.028 | 1.740 | 2.663 | -0.050 |
| 1.232 | 1.110 | 1.256 | -0.035 | 3.063 | 1.750 | 2.689 | -0.048 |

| X | PAL | RE | F | X | PAL | RE | F |
|---|---|---|---|---|---|---|---|
| 3.098 | 1.760 | 2.716 | -0.046 | 5.712 | 2.390 | 4.545 | 0.013 |
| 3.133 | 1.770 | 2.742 | -0.044 | 5.760 | 2.400 | 4.577 | 0.012 |
| 3.168 | 1.780 | 2.769 | -0.042 | 5.808 | 2.410 | 4.608 | 0.011 |
| 3.204 | 1.790 | 2.795 | -0.043 | 5.856 | 2.420 | 4.640 | 0.013 |
| 3.240 | 1.800 | 2.822 | -0.033 | 5.905 | 2.430 | 4.672 | 0.010 |
| 3.276 | 1.810 | 2.849 | -0.035 | 5.954 | 2.440 | 4.704 | 0.009 |
| 3.312 | 1.820 | 2.876 | -0.033 | 6.003 | 2.450 | 4.736 | 0.008 |
| 3.349 | 1.830 | 2.903 | -0.031 | 6.052 | 2.460 | 4.769 | 0.007 |
| 3.386 | 1.840 | 2.930 | -0.029 | 6.101 | 2.470 | 4.800 | 0.006 |
| 3.423 | 1.850 | 2.957 | -0.027 | 6.150 | 2.480 | 4.832 | 0.005 |
| 3.460 | 1.860 | 2.984 | -0.025 | 6.200 | 2.490 | 4.864 | 0.005 |
| 3.497 | 1.870 | 3.011 | -0.023 | 6.250 | 2.500 | 4.897 | 0.004 |
| 3.534 | 1.880 | 3.039 | -0.021 | 6.300 | 2.510 | 4.929 | 0.003 |
| 3.572 | 1.890 | 3.066 | -0.019 | 6.350 | 2.520 | 4.961 | 0.002 |
| 3.610 | 1.900 | 3.094 | -0.017 | 6.401 | 2.530 | 4.994 | 0.001 |
| 3.649 | 1.910 | 3.122 | -0.015 | 6.452 | 2.540 | 5.027 | 0.000 |
| 3.686 | 1.920 | 3.150 | -0.013 | 6.503 | 2.550 | 5.060 | -0.001 |
| 3.725 | 1.930 | 3.178 | -0.011 | 6.554 | 2.560 | 5.093 | -0.002 |
| 3.764 | 1.940 | 3.206 | -0.010 | 6.605 | 2.570 | 5.126 | -0.002 |
| 3.803 | 1.950 | 3.234 | -0.008 | 6.656 | 2.580 | 5.159 | -0.003 |
| 3.842 | 1.960 | 3.262 | -0.006 | 6.708 | 2.590 | 5.192 | -0.004 |
| 3.881 | 1.970 | 3.290 | -0.005 | 6.760 | 2.600 | 5.225 | -0.005 |
| 3.920 | 1.980 | 3.318 | -0.003 | 6.812 | 2.610 | 5.258 | -0.006 |
| 3.960 | 1.990 | 3.347 | -0.001 | 6.864 | 2.620 | 5.292 | -0.006 |
| 4.000 | 2.000 | 3.375 | 0.000 | 6.917 | 2.630 | 5.325 | -0.007 |
| 4.040 | 2.010 | 3.404 | 0.002 | 6.970 | 2.640 | 5.359 | -0.008 |
| 4.080 | 2.020 | 3.432 | 0.003 | 7.023 | 2.650 | 5.393 | -0.009 |
| 4.121 | 2.030 | 3.461 | 0.004 | 7.076 | 2.660 | 5.426 | -0.009 |
| 4.162 | 2.040 | 3.490 | 0.006 | 7.129 | 2.670 | 5.460 | -0.010 |
| 4.203 | 2.050 | 3.519 | 0.007 | 7.182 | 2.680 | 5.494 | -0.010 |
| 4.244 | 2.060 | 3.548 | 0.008 | 7.236 | 2.690 | 5.528 | -0.011 |
| 4.285 | 2.070 | 3.577 | 0.009 | 7.290 | 2.700 | 5.563 | -0.012 |
| 4.326 | 2.080 | 3.606 | 0.010 | 7.344 | 2.710 | 5.597 | -0.012 |
| 4.368 | 2.090 | 3.635 | 0.011 | 7.398 | 2.720 | 5.631 | -0.013 |
| 4.410 | 2.100 | 3.664 | 0.012 | 7.453 | 2.730 | 5.666 | -0.013 |
| 4.452 | 2.110 | 3.693 | 0.013 | 7.508 | 2.740 | 5.700 | -0.013 |
| 4.494 | 2.120 | 3.723 | 0.014 | 7.563 | 2.750 | 5.735 | -0.014 |
| 4.537 | 2.130 | 3.752 | 0.014 | 7.618 | 2.760 | 5.770 | -0.014 |
| 4.580 | 2.140 | 3.782 | 0.015 | 7.673 | 2.770 | 5.804 | -0.014 |
| 4.623 | 2.150 | 3.811 | 0.016 | 7.728 | 2.780 | 5.839 | -0.015 |
| 4.666 | 2.160 | 3.841 | 0.016 | 7.784 | 2.790 | 5.874 | -0.015 |
| 4.709 | 2.170 | 3.871 | 0.017 | 7.840 | 2.800 | 5.909 | -0.015 |
| 4.752 | 2.180 | 3.901 | 0.017 | 7.896 | 2.810 | 5.945 | -0.015 |
| 4.796 | 2.190 | 3.931 | 0.017 | 7.952 | 2.820 | 5.980 | -0.015 |
| 4.840 | 2.200 | 3.961 | 0.018 | 8.009 | 2.830 | 6.015 | -0.015 |
| 4.884 | 2.210 | 3.991 | 0.018 | 8.066 | 2.840 | 6.051 | -0.015 |
| 4.928 | 2.220 | 4.021 | 0.018 | 8.123 | 2.850 | 6.086 | -0.015 |
| 4.973 | 2.230 | 4.051 | 0.018 | 8.180 | 2.860 | 6.122 | -0.015 |
| 5.018 | 2.240 | 4.081 | 0.018 | 8.237 | 2.870 | 6.158 | -0.015 |
| 5.063 | 2.250 | 4.112 | 0.018 | 8.294 | 2.880 | 6.194 | -0.015 |
| 5.108 | 2.260 | 4.142 | 0.018 | 8.352 | 2.890 | 6.230 | -0.015 |
| 5.153 | 2.270 | 4.173 | 0.018 | 8.410 | 2.900 | 6.266 | -0.015 |
| 5.198 | 2.280 | 4.203 | 0.018 | 8.468 | 2.910 | 6.302 | -0.015 |
| 5.244 | 2.290 | 4.234 | 0.017 | 8.526 | 2.920 | 6.338 | -0.015 |
| 5.290 | 2.300 | 4.265 | 0.017 | 8.585 | 2.930 | 6.374 | -0.014 |
| 5.336 | 2.310 | 4.296 | 0.017 | 8.644 | 2.940 | 6.411 | -0.014 |
| 5.382 | 2.320 | 4.327 | 0.016 | 8.703 | 2.950 | 6.447 | -0.014 |
| 5.429 | 2.330 | 4.358 | 0.016 | 8.762 | 2.960 | 6.484 | -0.014 |
| 5.476 | 2.340 | 4.389 | 0.016 | 8.821 | 2.970 | 6.520 | -0.014 |
| 5.523 | 2.350 | 4.420 | 0.015 | 8.880 | 2.980 | 6.557 | -0.014 |
| 5.570 | 2.360 | 4.451 | 0.014 | 8.940 | 2.990 | 6.593 | -0.014 |
| 5.617 | 2.370 | 4.482 | 0.014 | 9.000 | 3.000 | 6.630 | -0.014 |
| 5.664 | 2.380 | 4.514 | 0.013 | | | | |
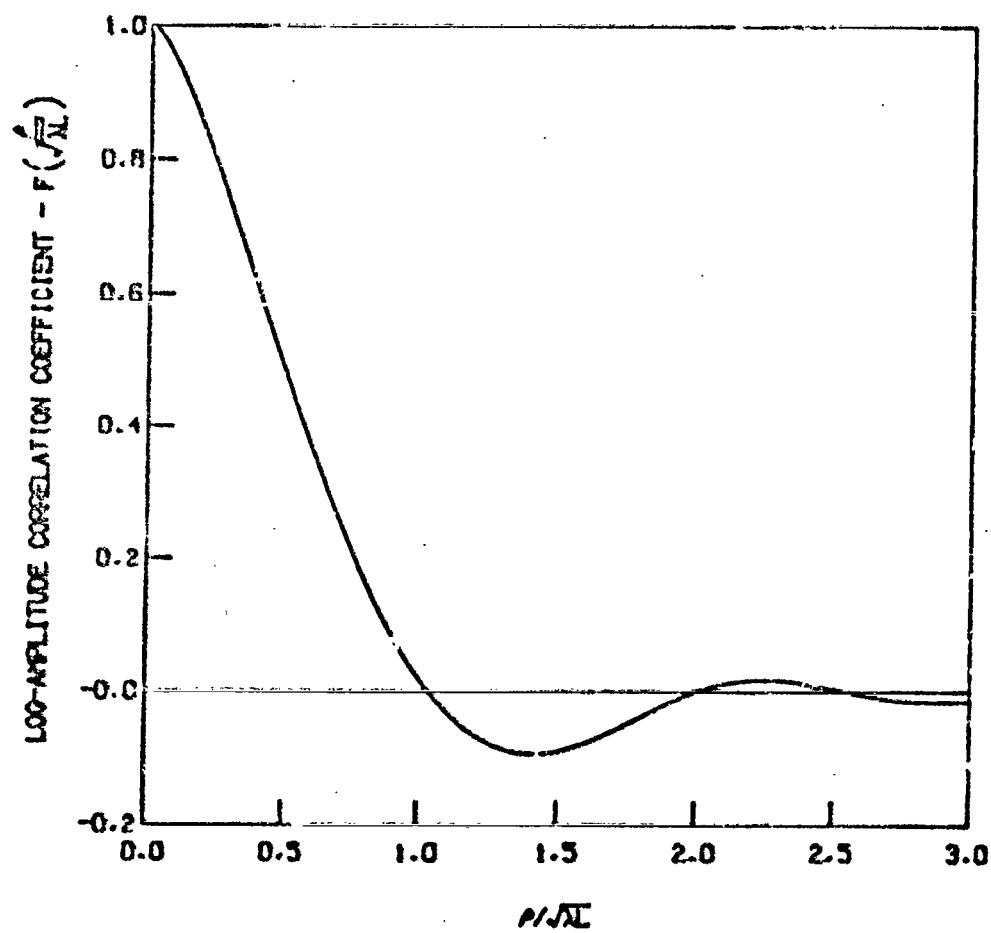
Figure B-1

# END

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